A Numerical Study on Mode Shapes of Point Supported Ferrocement Plates

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Abstract — Dynamic analysis is important for the structural design of the elements of the structures. A numerical study on the modes shapes of point supported ferrocement plates has been investigated under different support conditions, placement of point supports, moduli ratio of fibre and matrix, and mesh orientations using finite element formulation. For the analyis, a nine noded Lagrangian Mindlin plate element with enhanced shear interpolation is used. The analysis is performed for the first eight modes of vibration.

Keywords —Ferrocement, finite element, mode shape, point supports

I. Introduction

THE science and technology change very fast with time. The construction technology is not remained isolated from this change. Conventional reinforced concrete and ferrocement has been used for over a century and played the pioneering role in the early history of concrete construction. Ferrocement is a type of thin reinforced cement concrete where hydraulic cement is reinforced with layers of continuous and relatively small diameter wire meshes. In the past few years it has been increasing field application and laboratory research with this type of construction.

Although ferrocement can be considered as the modified form of reinforced concrete, there are differences between these. Ferrocement structural elements are normally thin thickness rarely exceeding 25 mm whereas for conventional reinforced concrete the thickness of structural materials normally exceeds 100 mm. Matrix in ferrocement mainly consists of Portland cement mortar except of coarse aggregate as in reinforced cement concrete. The reinforcement provided in ferrocement consists of large amount of smaller diameter wires or wire meshes and in reinforced concrete discretely placed reinforced bars are used having comparatively more diameter. In terms of structural behaviours, ferrocement exhibits very high tensile strength to weight ratio and superior cracking performance. Formwork is very rarely needed for the fabrication which conveys economical construction of certain structures such as geodesic domes, wind tunnels, circular storage structures and swimming pools.

Ferrocement consists of layered wire meshes and rich cement-sand mortar and has high degree of ductility and energy absorbing capacity. It has the advantage of high tensile strength to weight ratio and superior cracking performance over the conventional reinforced concrete [1]. It has the potential to be an attractive material for boats, barges, mobile homes or portable structures that must be light in weight and impact resistant.

An experimental study on shear capacity in flexure for simply supported ferrocement plates having a span/depth ratio of 2.0 was conducted with various wire mesh types as web reinforcement [2]. The flexural performance of ferrocement plates under normal and aggressive environments was investigated by M. Jamal [3]. The type of proposed work is not reported in the literature so far.

The response of ferrocement under static environment is well understood, although its behaviour under dynamic load continues to be unexplored. The aim of the present work is to investigate the mode shapes of the point supported ferrocement plate numerically under the effect of support conditions, placement of point supports, moduli ratio of fibre and matrix, and mesh orientations. A lot of cases of mode shapes for first eight mode has been studied, a few are presented here with 3D.

II. Governing Equations

The governing equations for a Mindlin ferrocement plate that incorporates transverse shear deformations are given below. The displacement field are given as

$$U(x, y, z) = z\,\theta_x(x, y)$$

$$V(x, y, z) = z\,\theta_y(x, y)$$

$$W(x, y, z) = w(x, y)$$

in which *x, y* are the rectangular coordinates in the plane of the plate, *z* is the thickness-direction coordinate measured downward from the mid-plane; *U, V* and *W* are the displacement in the *x, y* and *z* directions respectively, *w* is the corresponding mid-plane displacement, and θ_x and θ_y are the normal rotations in the *xz* and *yz* planes respectively due to bending.

The constitutive equation of a Mindlin plate showing stress-strain relationship can be written as

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{Bmatrix}$$

and

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$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} D_{44} & D_{45} \\ D_{45} & D_{55} \end{bmatrix} \begin{Bmatrix} \theta_x + w_{,x} \\ \theta_y + w_{,y} \end{Bmatrix}$$

where

$$D_{ij} = \int_{-h/2}^{+h/2} Q_{ij} z^2 dz \qquad i, j = 1, 2, 6$$

$$= K_{ij}^2 \int_{-h/2}^{+h/2} C_{ij} dz \qquad i, j = 4, 5$$

in which Q_{ij} (i, j =1, 2, 6) are the reduced in-plane stiffnesses for the plane stress, C_{ij} (i, j = 4, 5) are the appropriate elastic constant and K_{ij}^2 are shear correction factors.

To evaluate the rigidities of the Mindlin plate element, special orthotropy with respect to *x* and *y*-axis are considered. The plates are considered as composite laminate in Fig 1. The flexural rigidities are then given as

$$D_{ij} = \sum_{k=1}^{m} \int_{h_k}^{h_{k+1}} \overline{Q}_{ij}^{k} z^2 dz \qquad i, j = 1, 2, 6$$

$$= \sum_{k=1}^{m} \overline{Q}_{ij}^{k} \frac{\left(h_{k+1}^3 - h_k^3\right)}{3}$$

and shear rigidities are given as

$$S_{ij} = \sum_{k=1}^{m} \int_{h_k}^{h_{k+1}} \overline{C_{ij}} dz \qquad i, j = 4, 5$$

$$= \sum_{k=1}^{m} \overline{C_{ij}} \left(h_{k+1} - h_k\right)$$

where (h_{k+1} - h_k) - the thickness of the k th laminate and $\overline{Q_k}$ are obtained by suitably transforming the reduced in-plane stiffness for plane stress for each laminate.

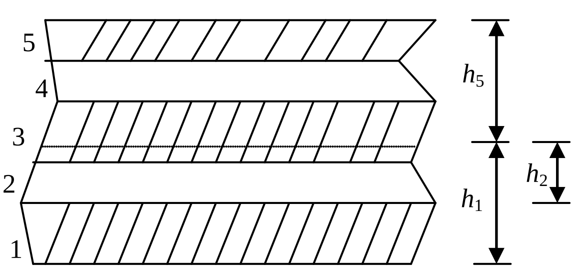


Fig. 1 Nomenclature for a typical ferrocement plate

The reduced in-plane stiffness for plane stress is given below.

$$Q_{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_{[k]}$$

where

$$Q_{11}^{k} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22}^{k} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{21}^{k} = Q_{12}^{k} = \frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66}^{k} = G_{12},\ C_{44}^{k} = G_{31},\ C_{55}^{k} = G_{23}$$

and E_{11}, E_{22} are the elastic moduli in the principal laminate direction 1 and 2,
ν_{12}, ν_{21} are the Poisson's ratio in the 1-2 plane,
G_{12} is the shear modulus in the 1-2 plane and
G_{13}, G_{23} are the transverse shear moduli in the transverse 3-1 and 2-3 planes

A. Finite Element Formulation

Consider the plate divided into nine noded Lagrangian elements introduced by Huang and Hinton [4]. Each node of the element has three degree of freedom w, θ_x, θ_y at each node. The total degrees of freedom of element are 27. The nodal displacement vector is presented by

$$\{d\}^T = [w_1\ \theta_{x1}\ \theta_{y1}\ w_2\ \theta_{x2}\ \theta_{y2} \ldots\ldots\ldots\ldots w_9\ \theta_{x9}\ \theta_{y9}]$$

Following the isoparametric element concept, the geometry of the element is given by as

$$x = \sum_{i=1}^{9} N_i x_i, \qquad y = \sum_{i=1}^{9} N_i y_i$$

The variations of displacement using same shape functions in an element is as follows

$$w = \sum_{i=1}^{9} N_i w_i, \qquad \theta_x = \sum_{i=1}^{9} N_i \theta_{xi}, \qquad \theta_y = \sum_{i=1}^{9} N_i \theta_{yi}$$

where w_i, θ_{xi} and θ_{yi} are the corresponding displacement and rotations at node i and N_i is the shape function associated with node i expressed in terms of the local coordinate system (ξ, η).

The elements of the vector $\{\varepsilon\}_p$ for curvatures and shear deformation are expressed in terms of the nodal displacements w_i, θ_{xi}, θ_{yi} . Thus it can be expressed as

$$\{\varepsilon\}_p = \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ \phi_x \\ \phi_y \end{Bmatrix} = [B] \begin{Bmatrix} w_1 \\ \theta_{x1} \\ \theta_{y1} \\ . \\ . \\ . \\ w_9 \\ \theta_{x9} \\ \theta_{y9} \end{Bmatrix}$$

or

$$\varepsilon_p = \sum_{i=1}^{9} [B_i]\{d_i\}$$

where displacement matrix

$$[B_i] = \begin{bmatrix} 0 & 0 & \frac{\partial N_i}{\partial x} \\ 0 & -\frac{\partial N_i}{\partial y} & 0 \\ 0 & -\frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial x} & 0 & N_i \\ \frac{\partial N_i}{\partial y} & -N_i & 0 \end{bmatrix} \quad [\ i = 1, \ldots, 9\ ]$$

and

$$\{d_j\} = \begin{Bmatrix} w_i \\ \theta_{xi} \\ \theta_{yi} \end{Bmatrix}$$

The stress resultant, $\{\sigma\}_p$ can be expressed in terms of nodal displacements after substituting for $\{\varepsilon\}_p$. Thus

$$\{\sigma\}_p = \begin{Bmatrix} M_X \\ M_Y \\ M_{XY} \\ Q_x \\ Q_y \end{Bmatrix} = [C]_p \{\varepsilon\}_p = [C]_p \sum_{i=1}^{9} [B_i]\{d_i\}$$

or

$$\{\sigma\}_p = [C]_p [B] \{d\}$$

where $[B] = \sum_{i=1}^{9} [B_i]$

The stiffness matrix of the element is given by

$$[K] = \iint [B]^T [C]_p [B] |J| drds$$

Gauss-Legendre rule is used for evaluating the stiffness matrix. It has been seen that Gauss-Legendre integration technique of 3×3 gives satisfactory results.

B. Free Vibration Analysis

Upon finite element discretization, the free vibration problem may be expressed in matrix form as

$$[K - \omega_i^2 M] a_i = 0 \qquad (i = 1, 2, \ldots\ldots. r)$$

The above equation is a typical eigen value problem. K is the global stiffness matrix, M is the global mass matrix, ω_i is the natural frequency, a_i is the ith mode shape and r is the total number of degree of freedom.

A typical submatrix of the consistent mass ***M***, which allows for the effects of rotary inertia and links nodes *i* and *j* of element having constant thickness can be expressed as

$$M_{ij}^{e} = C_{ij} \begin{bmatrix} h & 0 & 0 \\ 0 & \frac{h^3}{12} & 0 \\ 0 & 0 & \frac{h^3}{12} \end{bmatrix}$$

For lumped mass matrix ***M***, it is assumed that mass of element is lumped at discrete nodes of element. The lumped mass matrix is a diagonal matrix resulting in computational economy and is given by

$$M_{ij}^{e} = \frac{\rho h a}{3} \begin{bmatrix} 1 & 0 & . & . & 0 & 0 \\ 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & . & . & . & 1 & 0 \\ 0 & . & . & . & 0 & 1 \end{bmatrix} ..$$

III. Numerical Example

The typical slab tested by Clarke and Sharma [5] was used for the free vibration analysis and the details of the slab are given below as

Dimension of the slab (*a x b)* = 838.2 x 838.2 x 24.12 mm^3

Fibre modulus E_f = 62.87 kN/mm^2,

Matrix modulus E_m = 6.27 kN/mm^2

Fibre Possion's ratio ν_f = 0.3,

Matrix Possion's ratio ν_m = 0.2,

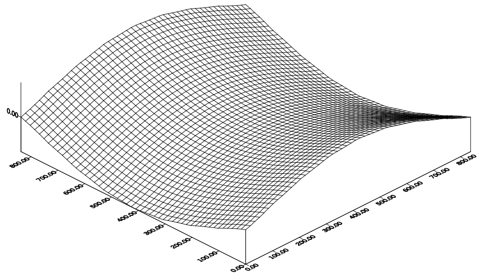
Fibre volume fraction V_f = 0.035,

Matrix volume fraction V_m = 0.965

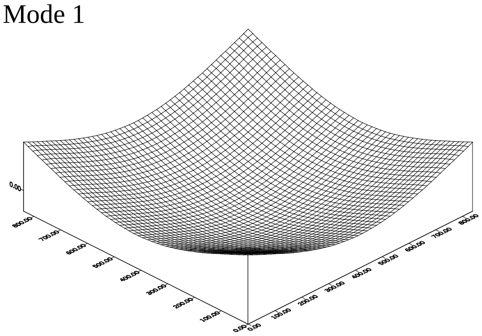
Ferrocement mass density per unit volume ρ = 0.24029 x 10^{-11} $kN\text{-}sec^2/mm^4$

Shear correction factor K = 0.833

The bending mode shapes for simple supported plates with placements of supports at 0.5 are given in Figures 2 for moduli ratio 1, mesh orientation $0^o$and lumped mass matrix.The computed natural frequencies having modular ratio of 1 and supports placed at dimensionless distance of 0.05 for lumped and consistent mass matrix are presented in Table 1. Table 2 list the natural frequencies obtained formodular ratio of 2 while supports were paced at a dimensionless distance 0.75.



Mode 1



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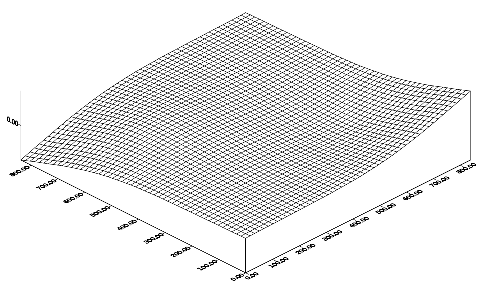

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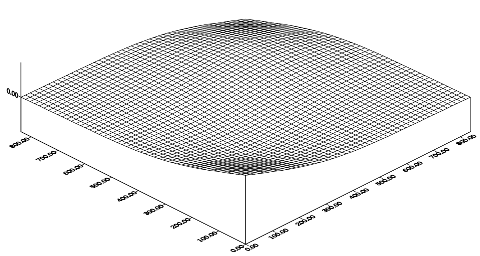

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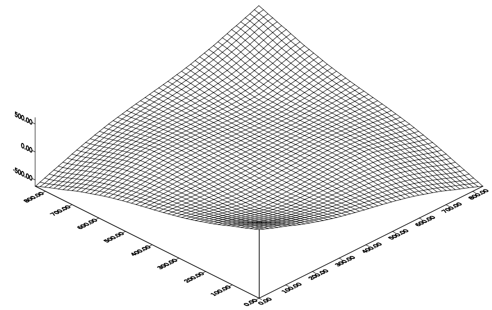

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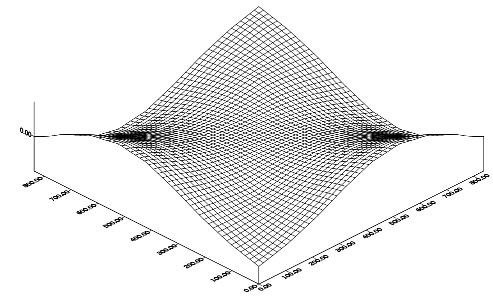

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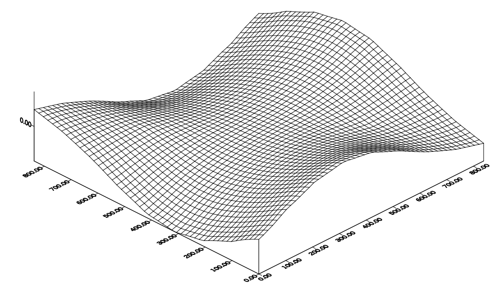

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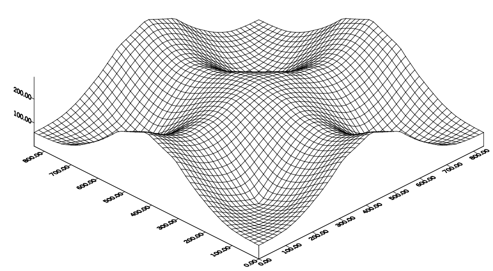

Mode 8

Fig. 2 Mode shapes for simple supports at 0.50, moduli ratio 1.0, and mesh orientation 0° for lumped mass matrix

TABLE 1
NATURAL FREQUENCY FOR MODULI RATIO 1.0 AND SUPPORTS AT 0.50

Orientation (°)	Modes	Simple supports		Fixed supports	
		Lumped (rad/s)	Consistent (rad/s)	Lumped (rad/s)	Consistent (rad/s)
0	1	385.65	386.41	652.65	653.87
	2	427.76	428.41	673.81	675.52
	3	590.43	591.46	698.00	699.33
	4	590.43	591.46	698.00	699.33
	5	624.29	624.99	721.34	722.21
	6	654.70	656.55	836.60	839.00
	7	654.70	656.55	836.60	839.00
	8	1027.29	1030.39	1071.87	1074.88
22.5	1	367.59	368.33	651.34	652.54
	2	424.58	425.32	659.87	661.60
	3	593.12	594.28	696.76	698.15
	4	593.12	594.28	696.76	698.15
	5	639.10	639.80	734.07	734.91
	6	657.05	658.82	843.02	845.42
	7	657.05	658.82	843.02	845.42
	8	1013.65	1016.77	1066.07	1068.79
45	1	348.20	348.09	644.78	646.54
	2	421.07	421.80	649.40	650.56
	3	589.27	590.58	692.60	694.04
	4	589.27	590.58	692.60	694.04
	5	653.09	653.79	746.03	746.83
	6	663.97	665.65	850.08	852.49
	7	663.97	665.65	850.08	852.49
	8	997.93	1001.07	1054.80	1057.40

TABLE 2
NATURAL FREQUENCY FOR MODULI RATIO 2.0 AND SUPPORTS AT 0.75

Orientation (°)	Modes	Simple supports		Fixed supports	
		Lumped (rad/s)	Consistent (rad/s)	Lumped (rad/s)	Consistent (rad/s)
0	1	213.18	213.38	440.35	441.28
	2	353.86	354.51	648.23	651.15
	3	415.86	416.40	788.61	792.57
	4	469.71	470.32	811.55	815.40
	5	809.48	820.16	987.20	993.58
	6	947.70	952.16	1259.61	1283.13
	7	1060.93	1075.49	1507.76	1532.74
	8	1141.14	1156.37	1615.23	1692.44
22.5	1	223.63	223.81	453.06	453.96
	2	347.20	347.86	677.84	680.86
	3	425.43	425.97	790.15	794.15
	4	463.41	464.00	793.72	797.32
	5	865.16	876.58	985.26	991.59
	6	952.05	956.80	1304.20	1327.71
	7	1098.34	1112.94	1477.91	1502.50
	8	1104.50	1119.59	1684.01	1761.06
45	1	235.16	235.34	465.60	466.50
	2	339.29	339.96	741.32	744.58
	3	445.99	446.56	741.32	744.58
	4	445.99	446.56	791.64	795.70
	5	956.16	960.77	983.05	989.33
	6	987.93	1000.99	1398.14	1421.99
	7	987.93	1000.99	1398.14	1421.99
	8	1186.64	1202.82	1765.45	1840.31

The analysis was performed for lumped mass matrix as well as consistent mass matrix for all parameter considered for the first eight modes. The plate was assumed to be consisting of five layers of lamina with each placed on another at the same direction for the evaluation of rigidities. The directions of mesh were taken as 0^o , 22.5^o , 45^o and moduli ratio as 1.0, 1.25, 1.50, 1.75 and 2.0. The fixed and simple supports are varied along diagonal with dimensionless distance of 0.25, 0.50 and 0.75 from the centre to corner of plate.

IV. CONCLUSION

The study on free vibration analysis of point supported ferro-cement plates under different support conditions, placement of point supports, moduli ratio of fiber and matrix, and mesh orientation using finite element formulation has been carried out for the first eight modes here. The analysis was performed for lumped mass matrix as well as consistent mass matrix. The supports were taken either simple or fixed. Some of the natural frequencies and mode shapes of vibration has been presented.

Ferrocement is a new composite construction material possessing enormous potential. A lot of research work is going on to unveil the static and dynamic properties completely. The present investigation can be extended further over the dynamic response of the ferrocement plate for combination of fixed and simple point supports, unsymmetrical point supports, supports provided at the points other than the diagonals of the plate, regardless of the number of supports, varying number of lamina in ferrocement plate, different mesh orientation for each lamina of ferrocement plate, varying aspect ratio of ferrocement plate, varying span to depth ratio of ferrocement plate,also the forced vibration analysis of point supported ferrocement plate is required to be investigated.

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