Thermo-Mechanical Behavior of Non Symmetric (Al/Al_2O_3) FGM Sandwich Plates

SAIDI Hayat[a], A. Tounsi[b] and EA. Adda Bedia[c]

Abstract— The analytical solution of non-symmetric functionally graded sandwich plates subjected to thermo-mechanical loading has been investigated by the use of a higher order shear deformation theory. This theory accounts for adequate distribution of the transverse shear strains in the thickness of the plate and satisfies the traction free boundary conditions on the top and bottom surface of the plates, thus a shear correction factor is not required.
The governing equations of equilibrium of non symmetric functionally graded sandwich plates can be obtained using virtuel work principle and the closed form solutions are obtained by using Navier technique. The validity of the present theory is demonstrated by comparison with solution available in the literature.
The results are presented for deflections and stresses of simply supported square plates. The influences played by of side to thickness ratio, volume fraction distribution, dimensionless ratio and slenderness ratio.

Keywords—sandwich plates; shear deformation, stress; thermo-mechanical;

I. INTRODUCTION

COPOSITE laminated materials are finding wide application in many engineering fields. Therefore, a number of approximate analytical and numerical methods have been developed in this area. Such composites are made of two or more materials to obtain a good structural performance, which the constituent does not show individually. Recently, advanced composite materials known as functionally graded material have attracted much attention in many engineering applications due to their advantages of being able to resist high temperature gradient while maintaining structural integrity [1]. The functionally graded materials (FGMs) are microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. They are usually made from a mixture of ceramics and metals to attain the significant requirement of material properties. Several studies have been performed to analyze the behavior of functionally graded plates and shells. Reddy [2] analyzed the static behavior of functionally graded rectangular plates based on his third-order shear deformation theory of plates.

There have been a considerable number of studies on the sandwich plates.

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Zenkour [3] presented a two-dimensional solution for bending analysis of simply supported functionally graded ceramic–metal sandwich plates. In his study, the sandwich plate faces are made of functionally graded material, the core layer is made of the isotropic ceramic material. Zenkour [4] studied the buckling and free vibration of the simply supported functionally graded sandwich plate. Woodward and Kashtalyan [5] presented the 3D elasticity solution for bending response of sandwich plates with functionally graded core. Zenkour and Alghamdi [6] studied the bending analysis of sandwich plates with functionally graded faces and homogeneous core under the effect of mechanical and thermal loads. Cinefra and Soave [7] presented the closed form solutions for free vibration of simply supported sandwich plate with isotropic faces and functionally graded core.

Saidi [8] studied thermo-mechanical bending response with stretching effect of functionally graded sandwich plates using a novel shear deformation theory.

In the present paper, analysis of non symmetric functionally graded sandwich plates under Thermo-mechanical loading is developed using a higher order shear deformation theory. This theory accounts for adequate distribution of the transverse shear strains in the thickness of the plate and satisfies the traction free boundary conditions on the top and bottom surface of the plates, thus a shear correction factor is not required.

The governing equations of equilibrium of non symmetric functionally graded sandwich plates can be obtained using virtuel work principle and the closed form solutions are obtained by using Navier technique. The validity of the present theory is demonstrated by comparison with solution available in the literature.

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Procedure for Paper Submission

II. PROBLEM FORMULATION

The present plate is composed of three layers, the top and bottom layers of non-symmetric sandwich plate is made of an isotropic homogeneous material ceramic and metal respectively. The core layer is made of an isotropic material with material properties varying smoothly in the thickness direction only, as shown in Figure1.

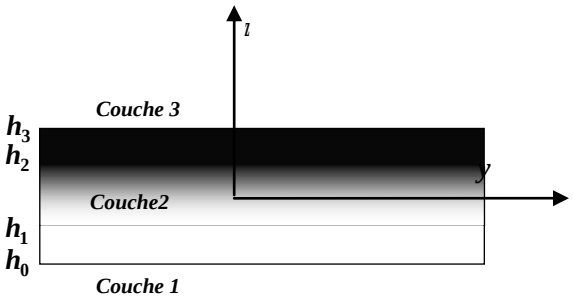

Fig.1 Geometry of the FGM sandwich pate.

The effective material properties for each layer, like Young's modulus and Poisson s ratio, can be expressed as:

$$P^{(n)}(z) = P_m + (P_c - P_m)V^{(n)} \tag{1}$$

Where Pm and Pc denote the property of the bottom and top faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$ (n = 1,2,3). Note that Pm and Pc are, respectively, the corresponding properties of the metal and ceramic of the FGM sandwich plate. The volume fraction.

III. GOVERNING EQUATIONS FOR FG SANDWICH PLATE

The displacement field for FG sandwich plates is described in the following equations Saidi [8]:

$$u(x,y,z) = u_0(x,y) - zw_{,x} + f(z)\theta_x \tag{2a}$$
$$v(x,y,z) = v_0(x,y) - zw_{,y} + f(z)\theta_y \tag{2b}$$
$$w(x,y,z) = w_0(x,y) + f(z)\theta_z \tag{2c}$$

Where, u, v, w are displacements in the x, y, z directions, u_0, v_0 and w_0 are midplane displacements, θ_x, θ_y and θ_z rotations of the *yz*, *xz*, and *xy* planes due to bending, respectively. $f(z)$ represents shape function determining the distribution of the transverse shear strains and stresses along the thickness and $f'(z) = \partial f(z)/\partial z$.

The linear strain expressions derived from the displacement model are as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + f(z)\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = f'(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{3}$$

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x},\ \varepsilon_y^0 = \frac{\partial v_0}{\partial y},\ \varepsilon_z^0 = \theta_z, \tag{4}$$

$$\gamma_{yz}^0 = \left(\theta_y + \frac{\partial \theta_z}{\partial y}\right),\ \gamma_{xz}^0 = \left(\theta_x + \frac{\partial \theta_z}{\partial x}\right), \tag{5}$$

$$\gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x},\ k_x = -\frac{\partial^2 w_0}{\partial x^2},\ k_y = -\frac{\partial^2 w_0}{\partial y^2}, \tag{6}$$

$$k_{xy} = -2\frac{\partial^2 w_0}{\partial x \partial y}, \tag{7}$$

$$\eta_x = \frac{\partial \theta_x}{\partial x},\ \eta_y = \frac{\partial \theta_y}{\partial y},\ \eta_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}. \tag{8}$$

The linear constitutive relations are given as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{9}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively, Q_{ij} are the stiffness coefficients.

The principle of virtual work, the following expressions can be obtained:

$$\int_{-h/2}^{h/2}\int_{\Omega}\left[\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right]d\Omega\,dz - \int_{\Omega} q\delta w\,d\Omega = 0 \tag{10}$$

The governing equations of equilibrium can be derived easily from Equation (10).

$$[C]\{\Delta\} = \{F\} \tag{11}$$

Where $\{\Delta\} = \{U,V,W,X,Y,Z\}^t$ and $[C]$ is the stiffness matrix,
$\{F\}$ is the force vector.

IV. RESULTS AND DISCUSSION

Numerous examples are solved to ensure the accuracy of the present theory for the prediction of static analysis under thermo mechanical loading , the closed form solution are obtained using the Navier solution of simply supported FGM sandwich plates.

Table1 contains the dimensionless center deflection $\bar{w}$ for an FG sandwich plate subjected to mechanical and thermal loads. The deflections are considered for $k = 0,1,2,3,4$ and 5 and different FG plates types:

[(0-1-0), (1-3-1), (2-1-2), (1-2-1), (1-0-1)].

It can be seen from the table1 that the results of the present theory are very close to those of the other shear deformation theories.

It can be observed that the HSDP overestimates the deflections comparatively to RHSDT and this, is due to the thickness stretching effect.

For a sandwich plate, the deflections increase with the power law index k .

TABLE I

DIMENSIONLESS CENTER DEFLECTIONS $\bar{w}$ OF THE DIFFERENT FG SANDWICH SQUARE PLATES.

k	*Theory*	*(0-1-0)*	*(1-3-1)*	*(2-1-2)*	*(1-0-1)*
0	**RHDPT**	**0.771340**	**0.891031**	**0.951864**	**0.959500**
	HDPT	0.817556	0.930218	0.983436	0.989882
	SSDPT	0.796783	0.906983	0.958491	0.964718
	TSDPT	0.808168	0.919718	0.972164	0.978509
	FSDPT	0.8957352	1.01729	1.07744	1.08468
1	**RHDPT**	**0.939609**	**0.951255**	**0.958373**	**0.959500**
	HDPT	0.989682	0.986027	0.989045	0.989882
	SSDPT	0.964682	0.961091	0.963918	0.964718
	TSDPT	0.978382	0.974764	0.977691	0.978509
	FSDPT	1.08353	1.07988	1.08371	1.08468
2	**RHDPT**	**0.974664**	**0.959818**	**0.959418**	**0.959500**
	HDPT	1.02533	0.994145	0.989945	0.989882
	SSDPT	0.999282	0.968891	0.964791	0.964718
	TSDPT	1.01355	0.982736	0.978573	0.978509
	FSDPT	1.12346	1.08928	1.08473	1.08468
3	**RHDPT**	**0.991418**	**0.962845**	**0.959755**	**0.959500**
	HYDPT	1.04333	0.997264	0.990245	0.989882
	SSDPT	1.01673	0.971882	0.965082	0.964718
	TSDPT	1.03131	0.985791	0.978873	0.978509
	FSDPT	1.14369	1.09287	1.08505	1.08468
4	**RHDPT**	**1.00309**	**0.964518**	**0.959900**	**0.959500**
	HDPT	1.05618	0.999082	0.990382	0.989882
	SSDPT	1.02920	0.973627	0.965209	0.964718
	TSDPT	1.04398	0.987582	0.979000	0.978509
	FSDPT	1.15807	1.09495	1.08519	1.08468
5	**RHDPT**	**1.01244**	**0.965664**	**0.959973**	**0.959500**
	HDPT	1.06654	1.00037	0.990445	0.989882
	SSDPT	1.03926	0.974864	0.965282	0.964718
	TSDPT	1.05421	0.988845	0.979073	0.978509
	FSDPT	1.16964	1.09644	1.08526	1.08468

The deflections of the FG sandwich and homogeneous plates decrease as a/h increases.

Figures 3 show the variation of the center deflection $\bar{w}$ with the aspect ratio a/b of sandwich plate type (1-3-1).

The deflection of the ceramic plate is found to be of the smallest magnitude and that of the metallic plate, of the largest magnitude. The increase of the aspect ratio a/b leads to a decrease of deflections of the homogeneous and FG sandwich plates.

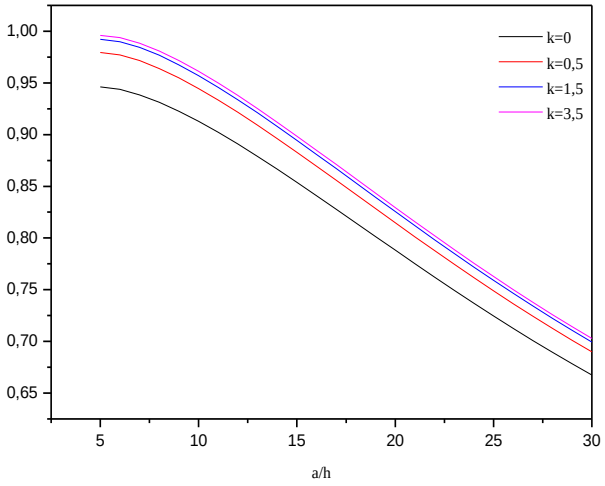

Fig. 2 Dimensionless center deflection $\bar{w}$ as a function of side-to-thickness ratio a/h for sandwich plate type (1-2-1)

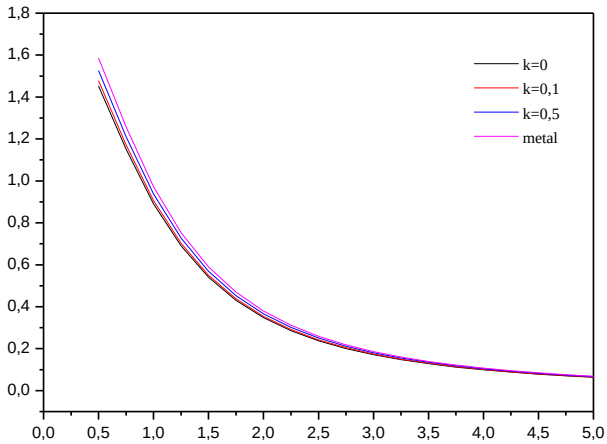

Fig. 3 Effect of the aspect ratio a/b on dimensionless center deflection $\bar{w}$ for sandwich plate type (1-3-1)

V. CONCLUSIONS

The applicability of the higher order shear deformation theory for the analysis of non symmetric functionally graded sandwich plate is assessed using closed form solution. The present theory incorporates the transverse shear stresses and strains and satisfies the conditions of zero transverse shear stresses and strains on the surface of the plate and hence eliminates the need of shear correction factor. The governing equations and boundary conditions are derived by employing the principle of virtual work. These equations are then solved via Navier –type solution. Numerical results are presented for severals types simply supported rectangular sandwich plates under thermo-mechanical loading. The effect of power- law index, span to thickness ratio and aspect ratio are studied.

It has been confirmed that the deflection of the sandwich plates decreases as the side to thickness and aspect ratios

increases. And the deflection increase as the power-law index increases.

The results show also that the including of stretching effect leads to decrease deflections and stresses.

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