Considering Soil Nonlinearity in 1D Simulation of Laterally Loaded Long Pile

Shivani Rani, and Amit Prashant

***Abstract*—** Pile is often simulated using Winkler type one-dimensional (1D) model using beam elements supported by a series of soil springs. Strain field in the soil around a pile subjected to lateral loads needs due consideration in defining the spring properties representing soil. It becomes a little more complex if one wants to incorporate soil nonlinearity. There are several relationships available in literature that correlate spring constant with secant modulus of soil. Some knowledge of Stiffness degradation with strain magnitude is also well documented in literature. This study explores various possibilities of combining these two sets of information to perform 1D analysis. The associated difficulties and required design assumptions are further discussed. A concept of equivalent lateral strain is proposed to determine applicable spring properties for equivalent-linear or nonlinear-1D analysis at different loads. These thoughts may help in developing simple tools for analysis and design of piles and other such structures.

***Keywords*—**1D model, Long Pile, Modulus Reduction Curve, Soil Nonlinearity, Spring Constant.

I. Introduction

A PILE, embedded in either homogenous or heterogeneous soil, is often subjected to lateral loads transferred through the supported structure. Due to horizontal movement of pile, the surrounding soil deforms and offers resistance to the movement or deflection of pile. The lateral strain evolved into the soil varies in both vertical and horizontal direction. Its magnitude is maximum at the ground surface and reduces with increasing depth. Below a certain depth the strain magnitude becomes negligible in case of long piles. Similarly, the lateral strain reduces with radial distance from pile and becomes negligible after a distance. Since soil is a nonlinear material, its stiffness decreases with increase in strain. Hence, soil is expected to have a significant variation in magnitude of its stiffness in both radial and vertical direction at deformed state of pile, i.e., the highest values at far distance from the pile top and the lowest values near the pile top where the load is transferred to pile. It is imperative to incorporate the lateral strain variation in order to estimate relatively more accurate response of soil-pile system. It can be reasonably simulated by performing 3D analysis using nonlinear material models for soils; however, it requires certain level of expertise and requires a lot of computational effort and time. Hence, 1D simulation is often preferred in design practice over 3D simulation and it requires estimation of lateral spring constant of soil that represents the resistance offered by the overall soil domain.

Since spring stiffness represents the stiffness of a wide region of soil, it does not incorporate the effect of shear transfer in tangential direction about pile as well as along the depth of pile. This remains as an approximation in the analysis and one can overcome its impact on the results by appropriately choosing the spring constant with due consideration to the pile-soil interaction.

The effect of stiffness reduction creates further complication in choosing the right value of spring constant. It may be possible to estimate modified properties of soil at different strain levels using one of the representative modulus reduction curves available in literature. However, the challenge is that the representative strain in soil is unknown when 1D analysis is performed. Nonlinear p-y curves are often used to simulate soil nonlinearity into the analysis of soil-pile system. These curves are supposed to correlate the pile displacement with applicable stiffness of spring. It is still difficult to estimate the representative p-y curves unless appropriate field experiments are performed to estimate these curves at different depths of soil, although a few empirical recommendations are available in literature. This article explores these issues in detail to look into the possibilities of using available relationships of modulus reduction curve to estimate spring constant or p-y curves instead of depending on expensive field experiments or using one of the grossly approximated p-y curve recommendations from literature.

II. Spring Constant In 1D Simulations

In design practices, 1D model is often used for predicting the response of soil-pile system over 3D model because it is simple, easy and less time consuming. Reference [15] concept is often used for 1D modeling according to which, soil-pile system can be considered as beam on foundation problem. In such a model, pile is simulated through beam elements and surrounding soil is modelled with springs. Here, springs are used to simulate the horizontal subgrade reaction offered by the soil due to application of lateral load. Since soil is a continuous medium, it is represented by providing a series of discrete springs at some spacing as shown in Fig. 1. For better results, the spacing between the springs should be kept

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minimum and the model should confirm displacement compatibility between the spring elements and beam elements at their connecting nodes.

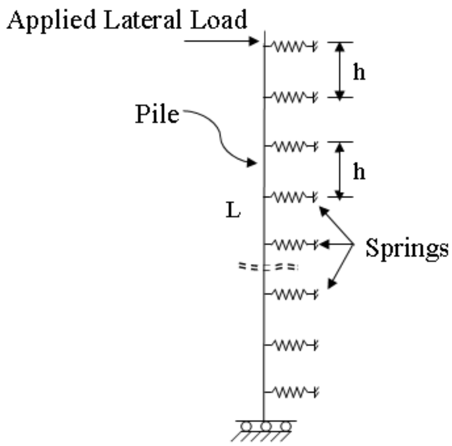

Fig. 1 Schematic representation of 1D model of a soil-pile system

Spring constant can be calculated through horizontal subgrade reaction provided by soil that can be estimated accurately by performing the plate load test in the field. This plate load test should be ideally performed in horizontal direction and at different depths. Another option is to perform a pressure meter test, which generates similar strain field into the soil. However, such tests are often expensive and difficult to perform at each location of pile in a project stretching to larger distances, e.g., long flyovers or elevated railway tracks. Alternatively, subgrade modulus can be approximated using the range of its values given by many researchers depending upon soil type, strength parameters of soil, etc. Many investigators also provided its correlation with SPT, CPT, PMT, etc test results. Some of these relationships between the horizontal subgrade reaction's coefficient and Secant Young's modulus of soil are listed in Table 1.

TABLE I
EMPIRICAL CORRELATIONS OF HORIZONTAL SUBGRADE REACTION WITH YOUNG'S MODULUS OF SOIL (ES)

S.No.	Author	Reference	Coefficient of Horizontal Subgrade Reaction
1	Terzaghi (1955)	[12]	0.74Es/D
2	Broms (1964)	[1], [2]	(0.48 - 0.90) Es/D
3	Muskhelishvili (1963)	[6]	2Es/D
4	Matlock (1970)	[5]	1.8 Es/D
5	Poulos (1971)	[7]	0.82 Es/D
6	Scott (1980)	[11]	Es/D

It is to be noted that a constant value has been used to correlate the horizontal subgrade reaction with the Young's modulus of soil in each case. In addition to that there is significant variation in those values, which will result into considerable deviation in the predicted values of spring constant using those equations. It is difficult to ascertain which one of these relationships will work the best for a particular site or a specific project. It is imperative to find a more fundamental method to estimate relatively accurate and project specific value of spring constant at different depths of soil profile that can provide reasonable prediction of pile deflection. Bending moment, shear force, slope, etc can be also obtained for a lateral load applied on pile.

III. STRAIN FIELD AROUND LATERALLY LOADED PILE

The soil-pile system deforms horizontally due to application of lateral load on a pile and generates a stress and strain field in the surrounding soil domain. Since the lateral stress transferred to the soil reduces along the pile depth and stress magnitude attenuates in radial direction, the strain varies in the both radial and vertical direction. The displacement pattern is shown in Fig. 2. It is maximum at the ground surface near the pile, whereas, it reduces along the depth as well as with the distance from the pile in radial direction.

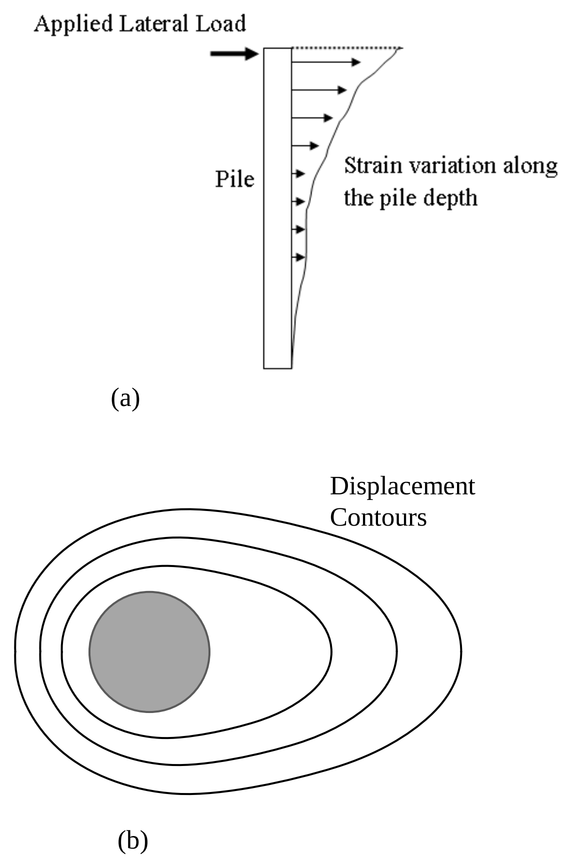

Fig. 2 Typical representation of displacement in soil (a) along the pile depth, and (b) on horizontal section at a depth

As shown in Fig. 2(a), soil displacement becomes almost negligible below certain depth, and hence the strain magnitude will be negligible too. The variation of strain is usually symmetric about the plane passing through pile axis and load vector if there is not much variation of soil properties in horizontal direction. Fig. 2(b) shows displacement contours on horizontal section at a depth. The contour values reduce with distance from pile. Wider contour gaps in the direction of loading indicate that a larger volume of soil goes through high strain in this direction in comparison to the other side. One may note here that the strain field in soil during pressuremeter test, with symmetric lateral loading about its axis, is slightly different from that of pile explained above. In fact, capturing the strain field of laterally loaded pile will be difficult through any available test except by performing pile load test itself.

IV. Design Practice

The deflection of soil-pile system should be estimated through the field test, i.e., pile load test; however, it involves a huge amount of cost and time. Hence, either 1D or 3D simulation is often performed to predict the deflection of pile-soil system. It is better to conduct 3D analysis in order to determine relatively good results for a soil-pile system; however, it requires some expertise to develop the model and involves significant effort and time in analysis. The complexity of 3D model may further increase if one has to incorporate nonlinearity associated with soil deformation and interface between soil and pile. Hence, simple and easy 1D models are generally used in routine practice since it reduces the complexities and time involved in 3D simulations.

For 1D modelling, it is important to use appropriate value of spring constant since estimate deflection is highly susceptible to that. The spring constant for linear analysis may be calculated using one of the recommendations available in literature (Table 1); however, significant variation in predicted values through these recommendations remains a challenge as explained in earlier section. Further, these recommendations require a representative value of Young's modulus and it is not clear whether one should use maximum Young's modulus corresponding to small strains or a secant modulus pertaining to some expected strain magnitude. Such uncertainties create confusion among design engineers.

V. P-Y Curves For Prediction Of Pile Deflection In Nonlinear Soil

A field tests, like PMT, DMT, etc may be performed at different depths of soil profile to predict the response of soil subjected to lateral loading. The measured pressure-displacement relationships can be then used to interpret the generic stress-strain relationships for soil at various depths. Since soil is a nonlinear material, it is expected that stress-strain relationship will be nonlinear. The shape of stress-strain curves depends upon the soil type. Nonlinear load-deformation relationship for soil reaction to pile deflection is often directly correlated with measured pressure-displacement relationships through experiments. Such load-deformation curves are referred to as p-y curves in 1D simulation of piles, which can be used in place of a spring constant, as shown in Fig. 3. It must be noted again though that the strain field in soil during experiments may be different from that due to laterally pile. Careful approximation may serve the purpose though.

Alternatively, the stress-strain relationship interpreted through these experiments may be correlated with expected relationship between soil reaction and deformation of pile. However, there is no such study available in literature. A typical stress-strain curve of soil and soil reaction-deformation curve are shown in Fig. 4. For a given boundary value problem, like laterally loaded long piles, it seems feasible to develop correlation between the curves in Fig. 4(a) and 4(b), and it is highly desirable to make research effort in this direction.

Since it is required to perform a number of field experiments along pile depth to accurately predict the behavior of soil-pile system, it becomes expensive in terms of both cost and time and generally not preferred in many projects. Hence, mathematical expressions, available in the literature, are often used to construct the nonlinear p-y curves.

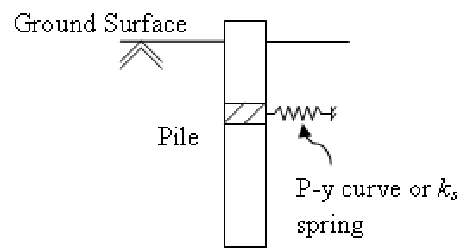

Fig. 3 Spring representing soil at a particular depth from the ground surface

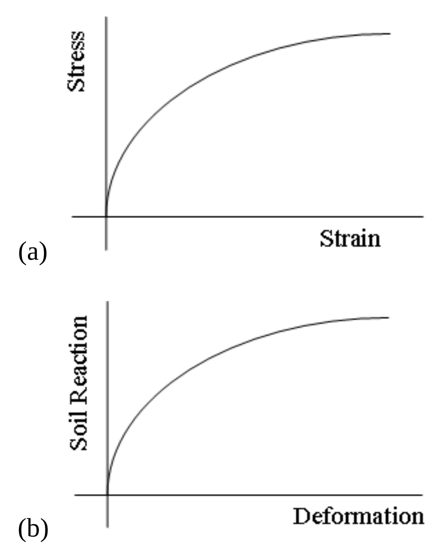

Fig. 4 Typical representation of (a) stress strain relationship, (b) soil reaction and deformation curve, for a soil profile

In the past, the procedure has been developed to construct p-y curves for both clay and sand. The pattern of curve depends upon the type of soil condition and loading, i.e., the shape of curve is different for sand below water table, sand above water table, clay below water table and clay above water table. Investigators, like [4], [8], [9], etc performed the field tests on the pile subjected to either static or dynamic loading and correlate their results with that from laboratory tests on undisturbed soil samples. Mathematical expressions were developed in order to fit the experimental results, and hence, provide the shape of p-y curves. Those expressions can be found easily in the literature. The expressions are the functions of y_{50}, i.e. strain at 50% of the maximum stress difference. UU triaxial compression tests were recommended to estimate y_{50}. This appears to be an approximate approach towards incorporating the nonlinearity, and hence needs more rigorous validation of recommendations. The effect of strain field needs to be given due consideration. The issues causing variation in recommended equations for spring constant in Table 1, which is much simpler, are applicable for such p-y recommendations too.

It may be interesting to correlate the p-y curves with complete stress strain curve with due consideration to the specific boundary value problem, like laterally loaded long pile. This approach may result into relatively better confidence in design. On the other hand, an attempt can be made to explore the procedure to use linear springs to predict the response of soil-pile system with nonlinear soil as discussed in next section.

VI. Linear Spring Constant For Nonlinear Soil To Perform 1D Analysis

The behaviour of soil is nonlinear and its stiffness decreases with increase in soil strain. P-y curves can be used to estimate the deflection of pile which can incorporate the nonlinearity of soil as explained in the previous section. One can also calculate the modified stiffness of soil at the expected lateral strain in soil. Hence, spring constant can be estimated using its correlations with secant modulus of soil at different strain level or at different lateral loads, and from which pile deflection can be determined. Fig. 5 shows deflection of pile top at the different load level considering different magnitudes of soil stiffness. It typically shows the pattern of deflection from 1D linear model at maximum value of spring constant and at its reduced values, i.e., k_1, k_2, k_3, and k_4, with lateral strain increase. However, it is difficult to determine the amount of expected strain level in soil that can be used to predict the spring constant of soil. It is also to be noted that the amount of reduction in soil spring constant with the strain level is also not known.

In the literature, many investigators, like, [3], [4], [10], [13], [14], etc showed the variation of shear modulus of soil with shear strain depending upon the soil type. The curves were generated by performing the dynamic laboratory tests like cyclic triaxial test, bender element test, resonant column test, etc. However, these can be used for static analysis as well, assuming that the so called the back-bone curve from dynamic testing results will coincide with the static stress-strain curve. Hence, one of those recommendations can be used to find the shear secant modulus at different strain levels. Fig. 6 shows a representative modulus reduction curve suggested by [3] for different types of soil conditions. Using this curve, modulus reduction factor can be obtained at different strain levels. For example, modulus reduction factor is estimated as 0.6 and 0.36 corresponding to 0.0003 and 0.001 values of strain. On the other hand, modulus reduction curve can be also developed by executing those tests on the undisturbed soil samples collected from the field. The curves developed through triaxial test can also be used in the analysis depending upon the level of accuracy achieved during the test. After determining the shear modulus of soil, Young's modulus of soil can be calculated from it at a value of soil's Poisson's ratio. Finally, spring constant can be estimated from Young's modulus of soil using any of the correlations listed in Table 1. The same concept can be then extended to develop correlations for p-y curves to perform non-linear 1D analysis.

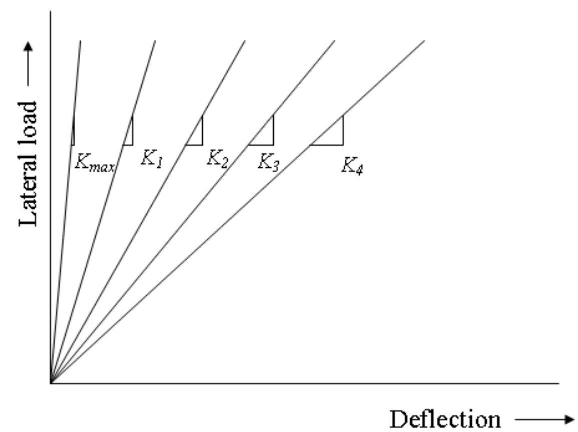

Fig. 5 Deflection of pile top at different magnitude of lateral load considering the different values strain level in soil

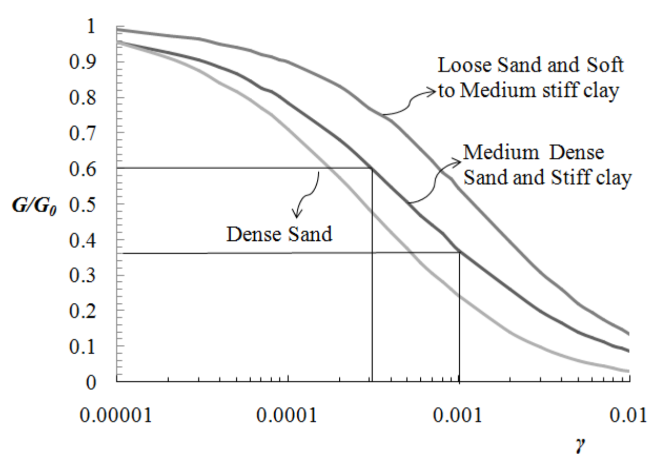

Fig. 6 Modulus reduction curve (after Imazu and Fukutake (1986)

The maximum soil modulus can be determined by using one of the recommendations available in literature based on SPT and CPT tests or by conducting geophysical survey of the site to do shear wave profiling. Once the maximum soil modulus is known, the soil modulus at different strain levels can be estimated using one of the representative modulus reduction curves. However, the amount of expected lateral strain in soil is still unknown. Further, the strain is expected to vary in both radial and vertical direction, which means the Young's modulus is varying too. Therefore, for a given relationship between spring constant and Young's modulus it is still difficult to estimate the applicable value of spring constant.

However, one can think of an equivalent lateral strain at a particular depth, which represents the strain field in some form and can be used to define the load-deformation relationship of spring. This equivalent strain can be then used to estimate the spring constant. It is expected that the equivalent lateral strain will be a function of lateral displacement of pile. Once such a relationship is established it is easy to find deflection of pile by employing an iterative approach for convergence of pile deflection at different assumed values of equivalent strain and corresponding stiffness of soil.

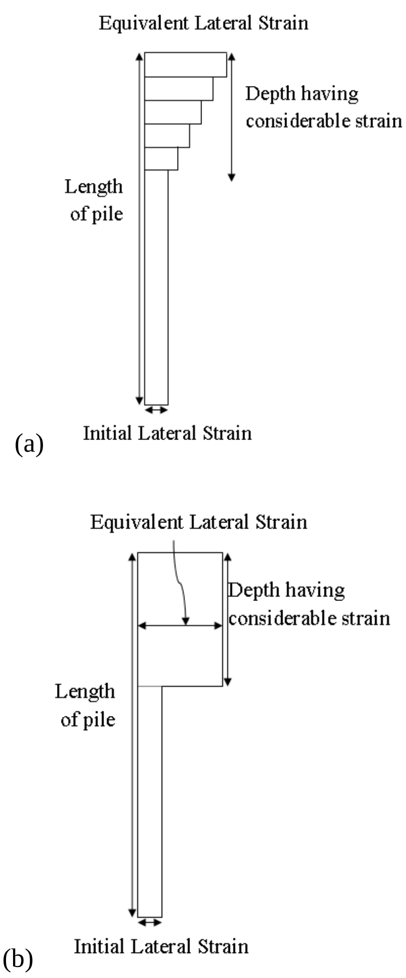

Fig. 7 Patterns of considered equivalent lateral strain (a) varying, and (b) constant along the depth of pile

The equivalent lateral strain can be assumed to vary along the depth of pile as shown in Fig. 7(a). This will require convergence of displacement at each depth of pile. For design simplification, one can assume a constant equivalent lateral strain along the depth of pile up to certain depth where strain is significant, as illustrated in Fig. 7(b). Below that depth, the effect of strain can be neglected and maximum value of spring constant can be used for the analysis. This equivalent lateral strain will represent the complete strain field up to the considered depth. Iterative procedure will become simpler though, since one can consider only the top deflection of pile for convergence. The authors are currently working on these possibilities to develop procedures for the estimation of equivalent lateral strain in soil for pile related problems. It is expected that such developments will help design engineers to have better confidence in assuming representative soil properties during simple yet more accurate simulations.

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