Simultaneous Optimization of Tolerances for Prismatic Part Assembly in Different Stack up Conditions

Dr. Dinesh Shringi, and Dr. Kamlesh Purohit

Abstract--- This paper presents a workable analytical model for finding the optimum set of tolerances and associated cost of prismatic part assembly that will minimize manufacturing cost and meet the imposed restraint conditions from different stack up conditions namely worst case (WS) and RSS. This new model is introduced based upon the empirical cost tolerance data of typical production process like rough milling and finish milling .In the present study, a new stochastic optimization algorithm based on hybridization of Simulated Annealing algorithm and Hook-Jeevs pattern search (SA-PS) has been used to solve the problem of tolerance optimization . The components of an assembly are manufactured using different machining operations. The process limits of the machine tool used for each operation puts limitation on the machining tolerances. Different machine tools have different process capability, thereby producing parts with different tolerances. This also leads to a set of necessary constraints for tolerance optimization.

Keywords-- tolerance, shaft-bearing assembly, synthesis, simulated annealing, pattern search

I. INTRODUCTION

TOLERANCE design is one of the most essential requirements of the part design and manufacturing. In practice, two types of tolerances are often defined: Design tolerance and manufacturing tolerance. The design tolerances are related to the functional requirements of a mechanical assembly or components, whereas manufacturing tolerances are used in process plan which must respect functional requirements as suggested by design tolerances. A common tolerance synthesis problem is to distribute the specified tolerance among the components of the mechanical assembly. This allocation of design tolerance among the components of a mechanical assembly and manufacturing tolerance to the machining process used in the fabrication of component plays a key role in the cost reduction and quality improvement.

Unnecessarily high tolerances lead to higher manufacturing cost while loose tolerance may lead to malfunctioning of the product. Traditionally, this important phase of product design and manufacturing is accomplished intuitively to satisfy design constraints based on past designs, standards, hand books, skills and experience of the designer and process planner. Tolerance design carried out by this approach does not necessarily lead to optimal allocation. Therefore, tolerance allocation has been widely studied in the literature. The review of the research carried by several researcher [1,3,5,6,7] presented reveals that in general tolerance design is carried out sequentially in two steps (i) functional (or design) tolerance allocation, and (ii) distribution of these tolerances on different manufacturing operations involving process capability of the machine, machining allowance etc. This sequential approach has serious limitations (i) infeasibility of design tolerance from the point of view of availability of manufacturing facilities. In this paper an attempt has been made to develop a model for the concurrent optimal synthesis of design and manufacturing tolerances for a given mechanical assembly. A non-linear, multi variable constrained optimization problem has been formulated with a view to allocate the tolerance for minimum manufacturing cost. The optimization problem has been solved using combined Simulated Annealing and Pattern Search (SA-PS) algorithm. The application of proposed methodology has been demonstrated through prismatic part assembly example.

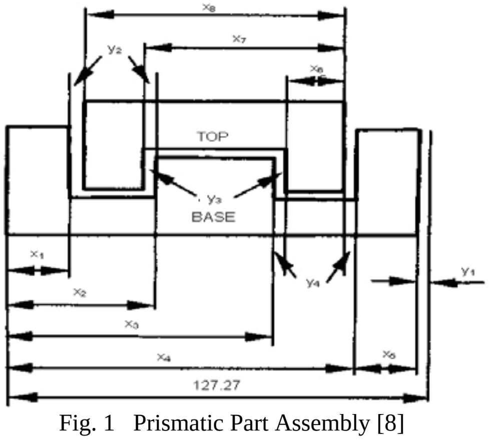

Fig. 1 Prismatic Part Assembly [8]

A prismatic part assembly consisting of base and top part is considered for optimal tolerance synthesis (Figure 1). This

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example has been taken from literature with some modification [8].

A. Assembly Response Function:

The above prismatic part assembly consists of eight-part feature dimensions (X_1, $X_2 \ldots X_8$) and four assembly response functions. The first assembly response function Y_1 is the machining allowance for the right surface of the base part. The other three response functions Y_2, Y_3, and Y_4 specify the clearance conditions between the two mating parts.
The assembly response functions Y_1, Y_2, Y_3, and Y_4 and eight feature dimensions are shown in Figure 1. Substituting the nominal dimensions of X_1 to X_8 in the assembly response functions (Eqs. 1to 3) give the nominal value of assembly response functions. Therefore, nominal value and tolerance of Y_1, Y_2, Y_3 and Y_4 are 0.127 ± 0.58 mm, 0.043 ± 0.28 mm, 0.025 ± 0.17 mm and 0.043 ± 0.23 mm [8]

II. PROCESS PLAN

It is assumed that both the base and top parts of prismatic assembly are initially available as die casted product, which needs machining. Both parts are machined with milling process. The machining process plans for both parts are:
(a) Base part – Rough milling – Finish milling
(b) Top part – Rough milling – Finish milling

III. FORMULATION OF OPTIMIZATION PROBLEM

A. Objective Function:

An objective function based on the minimization of the assembly manufacturing cost is formulated. Assembly manufacturing cost is obtained by summing up the cost of all the operations involved in manufacturing of assembly which include, cost of rough and finish milling of both base and top parts.

The manufacturing cost of an individual operation is represented as monotonically decreasing mathematical relationship between tolerance and the associated manufacturing cost. Several formulations have been evaluated / reviewed in the literature [4, 7, 8]. In this example a modified form of exponential cost function suggested by Dong [4], as expressed in the following equation, has been used. This cost function offers easier manipulation.

$$C(t) = C_0 e^{-C_1 t} + C_2 \qquad (5)$$

Where C_0, C_1 and C_2 are constants determined from the test data and found in the literature. The typical value of constants C_0, C_1 and C_2 for rough and finish milling operations are given in the Table I [1].

TABLE I
TYPICAL VALUES OF CONSTANTS, C_0, C_1 AND C_2 FOR PRISMATIC PART ASSEMBLY

Constants	Operations	
	Rough Milling	Finish Milling
C_0	92.84	160.43
C_1	13.66	86.70
C_2	1.72	29.20

Thus, the total cost of manufacturing an assembly can be expressed as

$$C = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij}\left(t_{ij}\right) \qquad (6)$$

IV. CONSTRAINTS

The objective function of the tolerance optimization is subjects to following constraints:

4.1 Stack-up condition constraints

The constraints on design tolerance are formulated based on two commonly used stack-up conditions, namely worst case (WC) and root sum square (RSS) are given below:

4.1.1 Worst Case stack up

$t_{41} + t_{51} \leq 0.58$ (7)
$t_{22} + t_{12} + t_{82} + t_{72} \leq 0.28$ (8)
$t_{72} + t_{62} + t_{32} + t_{22} \leq 0.17$ (9)
$t_{41} + t_{32} + t_{62} \leq 0.23$ (10)

4.1.2 Root sum square stack up

$t^2_{41} + t^2_{51} \leq 0.58^2$ ……… (11)
$t^2_{22} + t^2_{12} + t^2_{82} + t^2_{72} \leq 0.28^2$ (12)
$t^2_{72} + t^2_{62} + t^2_{32} + t^2_{22} \leq 0.17^2$ (13)
$t^2_{41} + t^2_{32} + t^2_{62} \leq 0.23^2$ (14)

4.1.3 Machining Constraints:

The constraints on the machining tolerances/ stock removal tolerance are based on Eq. (15). The sum of machining tolerance for an operation and that for its preceding operation should be less than or equal to the difference of nominal and minimum machining allowance for the operation. The nominal and minimum machining allowances are generally listed in the machining hand books [2]. This yields the following constraints:

$$t_{i1} + t_{i2} \leq \delta \qquad (15)$$

4.1.4 Process Limits Constraints

The process capability limits of a machine tool used for machining both the parts put the limitation on the Machining tolerances. For a general purpose milling machine the process limit tolerance for rough milling and finish milling operations may be assumed to vary between (0.08 – 0.5 mm) and (0.01 – 0.1 mm) respectively [8].

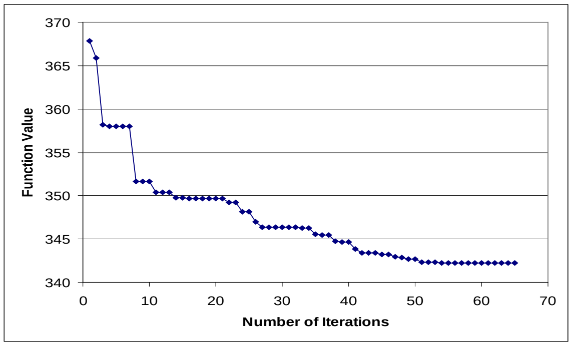

Fig. 2 Variation in Function Value

V. CONCLUSION

Optimized tolerance allocation for both rough milling and finish milling, corresponding to WC and RSS criteria, are reported in Table II. A close look to the Table II shows that WC criteria gives tight tolerances during finish milling operation where as RSS criteria gives loose tolerance. Hence the cost of manufacturing of prismatic assembly for RSS criteria is less than that of WC criteria. In rough milling operation the tolerance of almost all dimensions approaches to maximum process limits except in dimension X_4 and X_5 where no finish milling operation is being performed. The variation in objective function for 65 iterations (each temperature level is taken as one iteration) is shown in Figure 2. Initially at the high temperature, the objective

Function approaches to minimum at faster rate in first few iterations then it reduces gradually to the minimum value.

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TABLE II

OPTIMAL RESULTS FOR PRISMATIC PART ASSEMBLY

Criteria	Rough Milling		Finish Milling		Optimized Cost	Accumulated Tolerance
	Tolerance Notation	Tolerance (mm)	Tolerance Notation	Tolerance (mm)		
WC	t_{11}	0.5000	t_{12}	0.0949	$ 342.23	ΔY_1 =0.580 ΔY_2 =0.2791 ΔY_3 =0.1698 ΔY_4 =0.229
	t_{21}	0.5000	t_{22}	0.4448		
	t_{31}	0.5000	t_{32}	0.0409		
	t_{41}	0.1491	t_{42}	*		
	t_{51}	0.4309	t_{52}	*		
	t_{61}	0.5000	t_{62}	0.0399		
	t_{71}	0.5000	t_{72}	0.0442		
	t_{81}	0.5000	t_{82}	0.0959		
RSS	t_{11}	0.5000	t_{12}	0.0997	$ 319.80	ΔY_1 =0.5412 ΔY_2 =0.1970 ΔY_3 =0.1698 ΔY_4 =0.229
	t_{21}	0.5000	t_{22}	0.0980		
	t_{31}	0.5000	t_{32}	0.0703		
	t_{41}	0.2071	t_{42}	*		
	t_{51}	0.5000	t_{52}	*		
	t_{61}	0.5000	t_{62}	0.0709		
	t_{71}	0.5000	t_{72}	0.0963		
	t_{81}	0.5000	t_{82}	0.1000		

* The dimensions X_4 and X_5 are manufactured by rough milling operation only.