Mixed convection heat and mass transfer over a vertical plate in a power-law fluid with variable viscosity, Radiation and Soret effects

Pranitha Janapatla and Venkata Suman Gontla

***Abstract*—**The aim of this paper is to investigate the effect of variable viscosity on mixed-convection heat and mass transfer along a vertical plate. The study of mixed convection heat and mass transfer of non-Newtonian power-law fluid saturated Darcy porous medium with variable viscosity, Soret and radiation effects are analyzed. The nonlinear governing partial differential equations are transformed into ordinary differential equations using similarity transformations. These equations are solved numerically by using shooting technique. The numerical results for various values of the variable viscosity θ_e, radiation effect (R) and Soret (Sr) for velocity, temperature, concentration profiles are presented graphically. Local heat and mass transfer are shown in a tabular form.

***Keywords*—**Mixed convection, vertical plate, Porous media, Thermal boundary layer, Radiation effect, Soret effect, Variable viscosity, Power law fluid.

I. INTRODUCTION

In recent year, the heat transfer problem with a convective surface boundary condition has attracted the interest of many researchers, since it is more general and realistic especially in several engineering and industrial processes such as transpiration cooling process, material drying, polymers, cosmetics and toiletries, laser pulse heating, ground water flow etc. A review of convective heat transfer in porous medium is presented in the book by Nield and Bejan [1]

The scope of the current research is to implement more appropriate models for power law fluids properties and study the effect of these models on heat and mass transfer in mixed convection. Srinivasacharya and Swamy Reddy [2] have studied the effect of thermal radiation and chemical reaction on a mixed convection from a vertical plate in a power-law fluid saturated Darcy porous medium. Srinivasacharya et al. [3] has been investigated numerical solution to free convection over a vertical plate saturated in a power-law fluid magnetic and double dispersion effects.

The effect of variable viscosity is considered for mixed convection along a vertical plate embedded in a saturated porous medium. The limiting cases of natural and forced convection are also examined. Similarity solutions are obtained for an isothermally heated plate with fluid viscosity varied as an inverse function of temperature. It is well known that the viscosity of liquid changes considerably in temperature, this influence the variation of the velocity, temperature through the flow. For example,enhancement in temperature from $10^0 c(\mu = 0.031 g/cms)$ to $50^0 c(\mu = 0.00548 g/cms)$ causes the diminishment in the viscosity of water by 240 % have investigated by Ling and Dybbs [4]. Horne and sullivan [5] have studied the geophysical convective system, the fluid at the core of the earth serves at a very high temperature and the earth's crust as a porous medium. The fluid embedded in the crust is subjected to a very high temperature of about 250 K, as one goes deeper into the interior of the earth and the upper surface is at a lower temperature. Therefore the viscosity of the fluid should be taken as a variable quantity, which depends on the temperature of the fluid. Further, the analysis of convection through porous media with temperature-dependent viscosity is important in several engineering applications such as cooling of nuclear reactors, food processing, petroleum reservoir operations, casting and welding in manufacturing processes, etc. Pal and Mondal [6] analized the mhd non-Darcy mixed convective diffusion of species over a stretching sheet embedded in a porous medium with non-uniform heat source/sink, variable viscosity and Soret effect.

Heat and mass transfer with thermo-diffusion effect is a subject of intensive research due to a wide range of applications in engineering and technology. These include geophysics, multi-component melts, oil-reservoirs, isotope separation, and in mixture between gases. Kairi and Murthy [7] have reported the Soret effect on free convection over a melting vertical flat plate in a non-Darcy porous medium saturated with a fluid of variable viscosity. Bennacer et al.[8] have studied both numerically and analytically the Soret effect on convection of a binary fluid saturating a horizontal porous layer. It was found that the analytical solution is in good agreement with the numerical solution of the full governing equations. Shateyi et al.[9] considered the problem of magnetic field on mixed convection from vertical surface in porous medium with effects of thermal radiation, hall currents, Soret and dufour.

Radiation effect on the mixed convection flow has very important application in physics and engineering particularly in space technology and processes involving high temperature. Recently, thermal radiation has been using the hyper sonic flights, missile reentry, rocket combustion chambers, power plants for interplanetary flight, gas cooled nuclear reactors, high temperature plasmas and underground nuclear waste disposal are some important applications. Aydin and Kaya [10] have investigated effects of thermal radiation on a steady MHD mixed convective flow of a viscous incompressible fluid about a permeable vertical plate. The effect of radiation on a mixed convection boundary-layer flow over an isothermal vertical wedge embedded in a porous medium saturated with a nanofluid is studied [11] by chamkha et al. Hossain et al.[12] have focused on

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the natural convection flow with variable viscosity and thermal radiation effect from a porous vertical plate. Grosan and Pop [13] have performed the effect of radiation on the mixed convection flow in a vertical channel for laminar and fully developed flow.

The main purpose of the present investigation is to illustrate the effect of variable viscosity, thermal diffusion and thermal radiation on mixed convection heat and mass transfer from a vertical plate in a power-law fluid saturated porous medium using the similarity transformations.

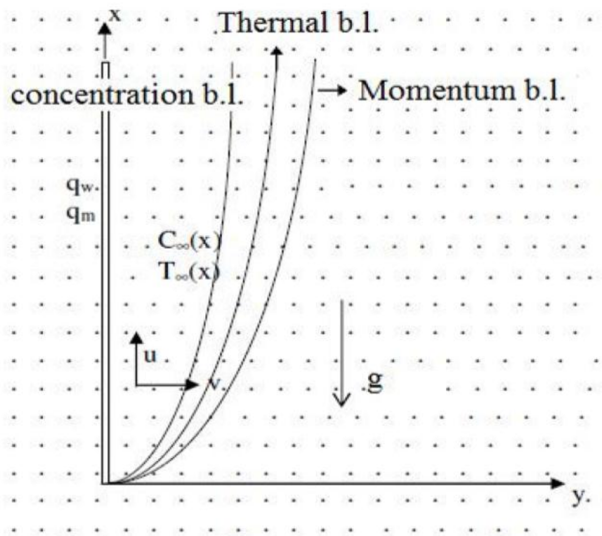

Figure 1: Physical model and coordinate system

II. Mathematical Formulation

We consider the mixed convection heat and mass transfer boundary layer flow over the vertical plate in a non-Newtonian power-law fluid saturated porous medium scheme is shown fig (1). Choose the two dimensional coordinate system such that the x-axis is along the vertical plate and y-axis normal to the plate. The plate is maintained with temperature and concentration, Tw and Cw respectively.

The Boussinesq approximation and making use of the standard boundary layer approximation, the governing equations for the power-law fluid are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u^n=\{U(x)\}^n+\frac{gK\rho_\infty}{\mu}(\beta_T(T-T_\infty)+\beta_C(C-C_\infty)) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left[\alpha_m\frac{\partial T}{\partial y}-\frac{1}{\rho C_p}q_r\right] \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_mK_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions are

$$\left.\begin{array}{l} v=0,\ T=T_w(x),\ C=C_W(x)\ \ at\ \ y=0 \\ v=U(x),\ T=T_\infty(x),\ C=C_\infty(x)\ \ at\ \ y\to\infty \end{array}\right\} \tag{5}$$

Here x and y are the cartesian coordinates, u and v are the average velocity components in x and y directions, respectively, T is the temperature, C is the concentration, β_T and β_C are the thermal and concentration expansion coefficients respectively, ν is the kinematic viscosity of the fluid, K is the permeability, K_T is the thermal diffusion ratio, α_m and D_m are the thermal and mass diffusivities of the porous medium,Cp is the specific heat capacity, q_r is the radiative heat flux term,and n is the powerlaw-index. where the subscripts w and ∞ indicate the conditions at the plate at some reference point in the medium and at the outer edge of the boundary layer respectively.

The radiative heat flux q_r is described by the Rosseland approximation such that

$$q_r=\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

Where σ^* and k^* are the Stefan-Boltzmann constant and the

$$T^4=4T_\infty^{\,3}T-3T_\infty^{\,4} \tag{7}$$

We assume now that the viscosity μ of the fluid-saturated porous medium depends on the temperature T in the following form

$$\frac{1}{\mu}=\frac{1}{\mu}\left[1+\gamma(T-T_\infty)\right]=a(T-T_e) \tag{8}$$

Where a,μ_∞,γ and T_e are constants,with a and T_e given by $a=\frac{\gamma}{\mu_\infty}$

and
$$\left.\begin{array}{l} \theta_e=\dfrac{T_e-T_\infty}{T_w(x)-T_\infty}=-\dfrac{1}{a(T_w-T_\infty)} \\ (\theta-\theta_e)=\dfrac{T-T_e}{T_w(x)-T_\infty} \end{array}\right\} \tag{9}$$

The continuity Eq.(1) is satisfied by introducing a stream function $\psi(x,y)$ such as

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \tag{10}$$

Introducing the following non-dimensional variables

$$\left.\begin{array}{l} \eta=By,\ \ \psi=Axf(\eta) \\ \theta(\eta)=\dfrac{T-T_\infty}{T_w(x)-T_\infty},\ T_w(x)-T_\infty=Ex^n \\ \phi(\eta)=\dfrac{C-C_\infty}{C_w(x)-C_\infty},\ C_w(x)-C_\infty=Fx^n \end{array}\right\} \tag{11}$$

and substituting Eq.(11) into Eq.(2)-(4) we obtain

$$(f')^{n} = 1 + (\theta + N\phi)(1 - \frac{\theta}{\theta_e}) \quad (12)$$

$$\left[1 + \frac{4}{3}R\right]\theta'' = (nf'\theta - f\theta') \quad (13)$$

$$\phi'' = Le[nf'\phi - f\phi' - Sr\theta''] \quad (14)$$

Where the prime denotes differentiation with respect to η, $R = 4\sigma^* T_\infty^3 / k^* k$ is the conduction radiation parameter, $N = \beta_c (C_w - C_\infty) / \beta_T (T_w - T_\infty)$ is the Buoyancy ratio, $Le = \alpha_m / D_m$ is the Lewis number, $A = (EgK\beta_T \alpha_m^n / \nu)^{1/2n}$ and $B = (EgK\beta_T / \nu\alpha_m^n)^{1/2n}$.

The boundary conditions Eq.(5) becomes

$$\left.\begin{aligned} f(0) = 0,\ \theta(0) = 1,\ \phi(0) = 1 \\ f(0) = 1,\ \theta(\infty) = 0,\ \phi(\infty) = 0 \end{aligned}\right\} \quad (15)$$

Results of practical interest are both heat and mass transfer rates. The local Nusselt number (Nu), and the local Sherwood number Sh, are, respectively given by:

$$\left.\begin{aligned} \frac{Nu_x}{x} &= -\theta'(0)(1 + \frac{4R}{3}) \\ \frac{Sh_x}{x} &= -\phi'(0) \end{aligned}\right\} \quad (16)$$

A. *Results and discussion*

The transformed ordinary differential equations, with the corresponding boundary conditions, are solved numerically using the Runge-Kutta fourth order and by the shooting technique by giving appropriate initial guess values of $f'(0)$, $\theta(0)$ and $\Phi(0)$ to match the values with the corresponding boundary conditions at $f(\infty)$, $\theta(\infty)$, $\phi(\infty)$ respectively.

The numerical results of velocity component $f'(0)$, temperature $\theta(0)$ distribution and concentration $\Phi(0)$ have been obtained for various values of the power-law index 'n' taking values 0.5 and 1.5. The radiation parameter is taken in the range of 0 to 1.5, the Soret parameter is taken in the range of 0 to 1.5 and the variable viscosity is taken in the range of 1.5 to 3.

We have investigated the effect of the variation of radiation parameter of non-dimensional velocity, temperature and concentration profiles in fig.2-7 for fixed values of N=1, Le=1, $\theta_e = 2$, Sr=0.5. fig .2-4 are the power-law index pseudo-plastic fluid n=0.5 and fig. 5-7 are for dilatant fluid with n=1.5. It is observed from the fig. 2 and fig. 5 that the velocity near the plate reduces but increases away from the plate while increasing the values of radiation (R). Figure.3 and fig. 6 depicts that the non-dimensional temperature increases as the radiation parameter (R) increases. It is observed from fig. 4 and fig. 7 that the concentration decreases near the vertical plate and increases away from the plate with increase in the radiation parameter (R).

The Soret parameter (Sr) on the non-dimensional velocity, temperature and concentration profiles are depicted in fig. 8 - 13 for fixed values of the parameters N=1, Le=1, R=0.5, Sr=0.5, $\theta_e = 2$. fig. 8-10 are the pseudo-plastic fluid n=0.5 and fig. 11-13 are the dilatant fluid with n=1.5. Fig.8 and fig. 11 shows that the velocity profile increase with an increase in the sort parameter. It is clear from fig. 9 and fig.12 that the temperature of the fluid increases with an increase in the Soret parameter. It is seen that the concentration decreases very slightly with increasing the Soret parameter in fig 10 and fig. 13.

The fig 14 - 16 Shows that the non-dimensional velocity, temperature, concentration profiles for fixed N=1, Le=1, R=0.5, Sr=0.5 on the variable viscosity by considering the power-law index n=0.5 and n=1.5. Figure 14 and fig.17 illustrates that while increasing the values of the variable viscosity the velocity increases the near the plate and decreases away from the plate. It can be seen from figure 15, 16, 18 and 19 that the temperature and the concentration profiles in the medium slightly decreases with an increase in the value of the variable viscosity.

Table 1 describes the effect of the local Nusselt number and local Sherwood number for different values of power-law index, radiation, Soret and variable viscosity parameters.

It is observed that in increasing the value of power-law index the heat and mass transfer coefficient decreases. Higher the values of radiation parameter enhances the heat and mass transfer coefficient. An increase in the value of Soret parameter increases the heat transfer coefficient, but decrease in mass transfer coefficient. An increase in the variable viscosity increases the heat transfer coefficient, but decrease in mass transfer coefficient.

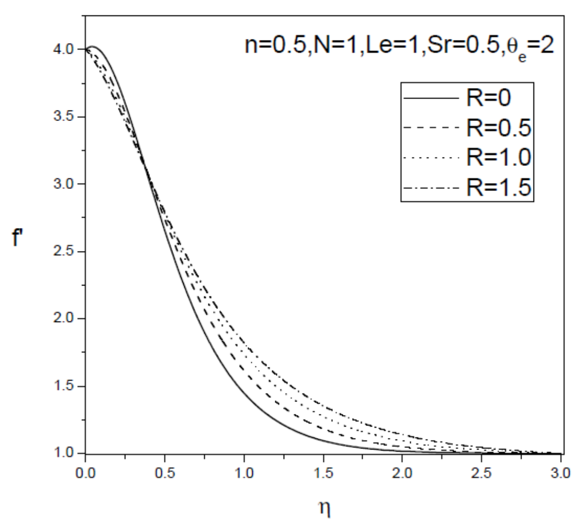

Figure 2. Velocity profile for various value of R for pseudo-plastic fluid.

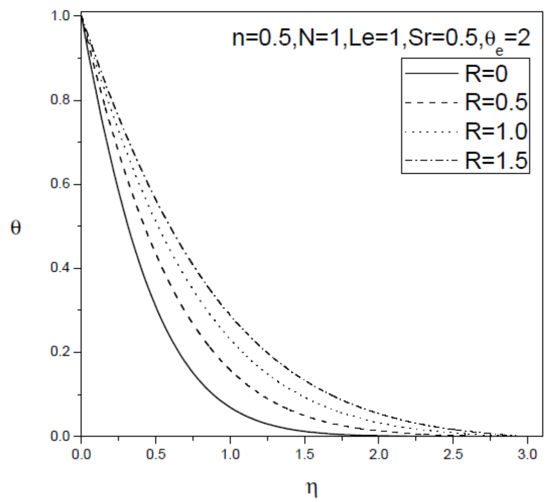

Figure 3. Temperature profile for various value of R for pseudo-plastic fluid

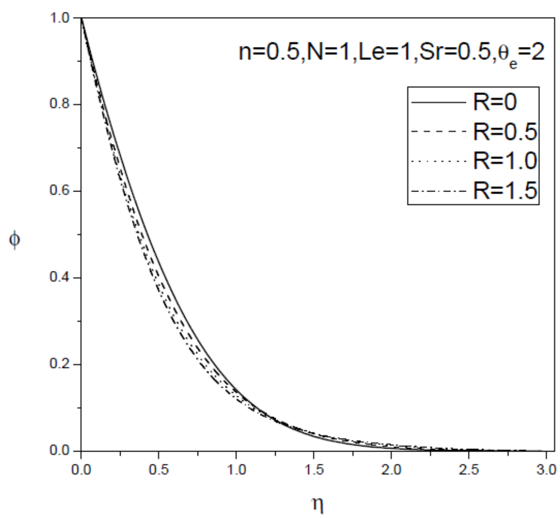

Figure 4.Concentration profile for various value of R for pseudo-plastic fluid.

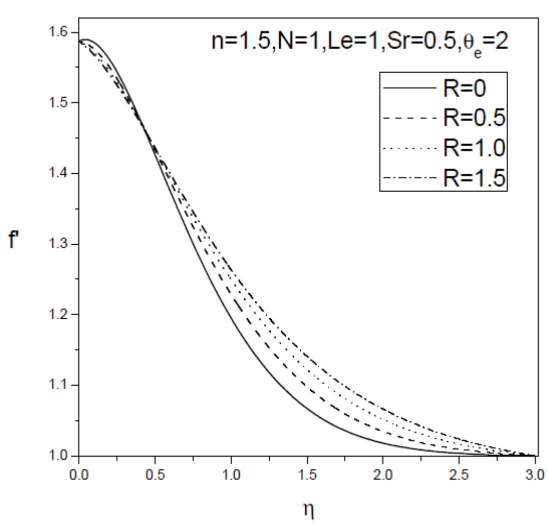

Figure 5. Velocity profile for various value of R for dilatant fluids fluid.

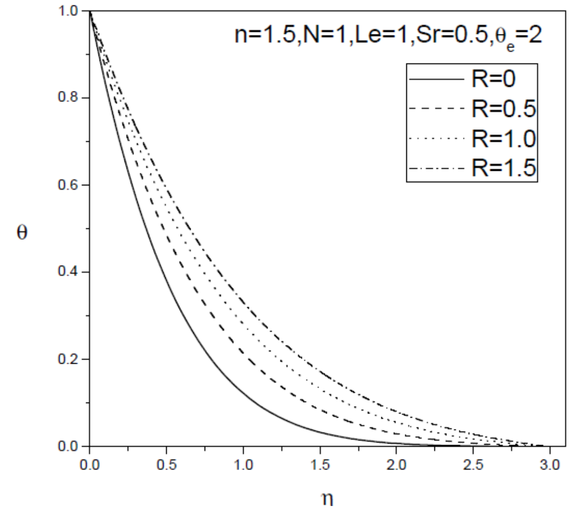

Figure 6. Temperature profile for various value of R for dilatant fluids fluid.

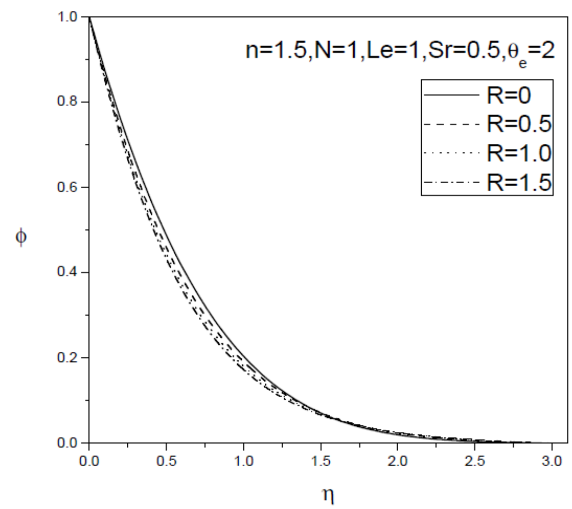

Figure 7. Concentration profile for various value of R for dilatant fluids fluid

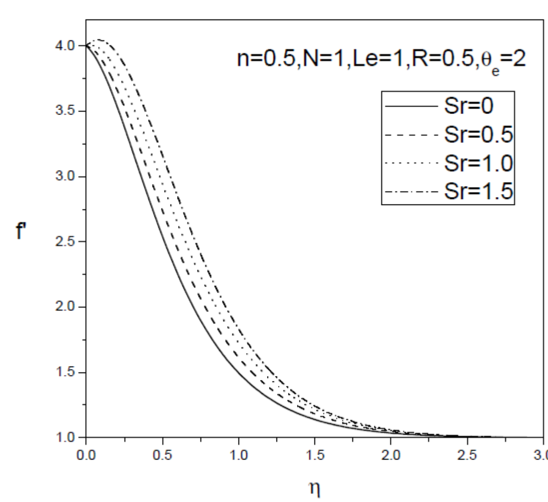

Figure 8. Velocity profile for various value of Sr for pseudo-plastic fluid.

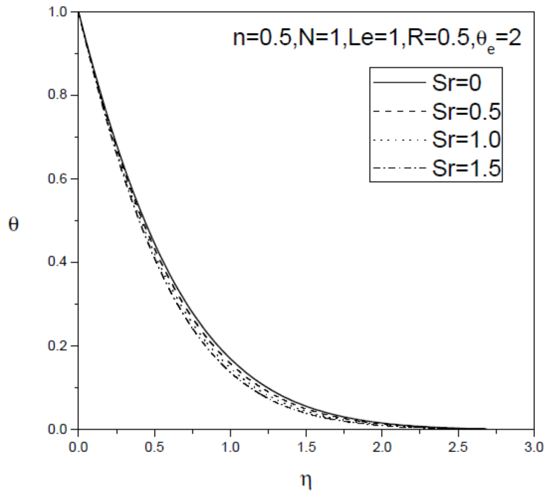

Figure 9. Temperature profile for various value of Sr for pseudo-plastic fluid.

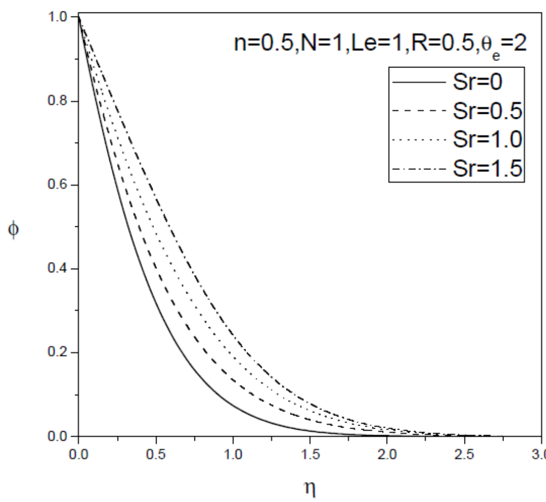

Figure 10.Concentration profile for various value of Sr for pseudo-plastic fluid.

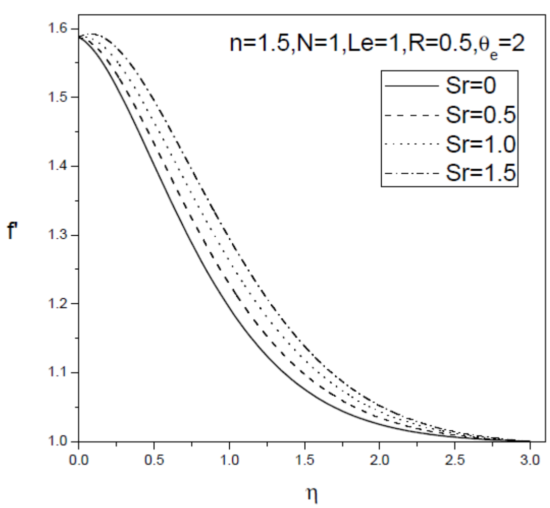

Figure 11. Velocity profile for various value of Sr for dilatant fluids fluid

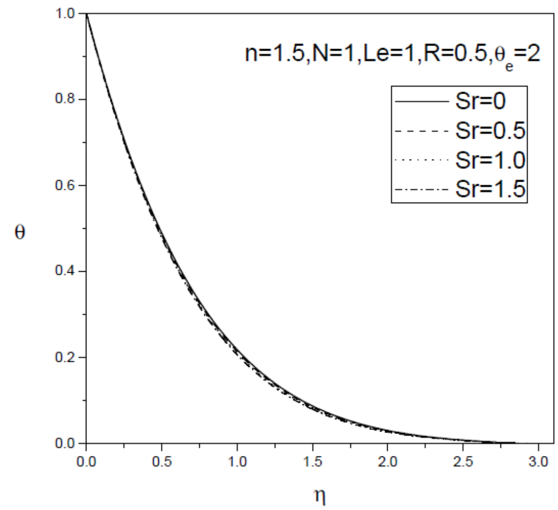

Figure 12. Temperature profile for various value of Sr for dilatant fluids fluid

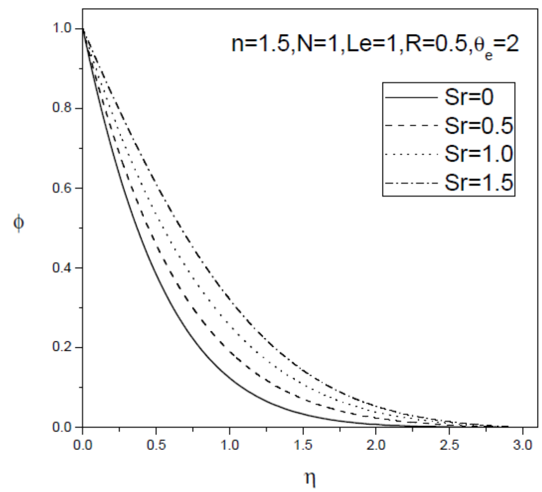

Figure 13. Concentration profile for various value of Sr for dilatant fluids fluid

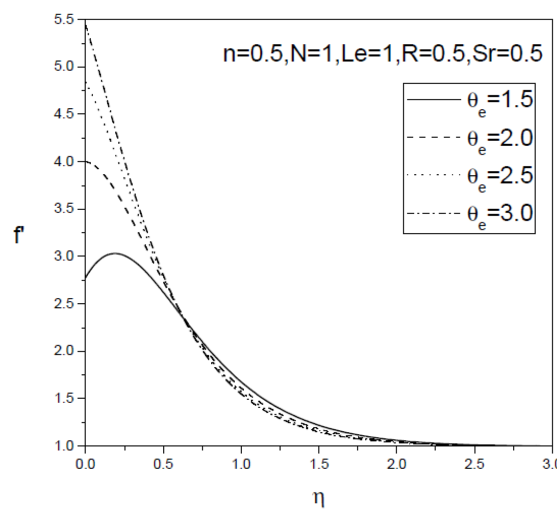

Figure 14. Velocity profile for various value of for θ_e pseudo-plastic fluid.

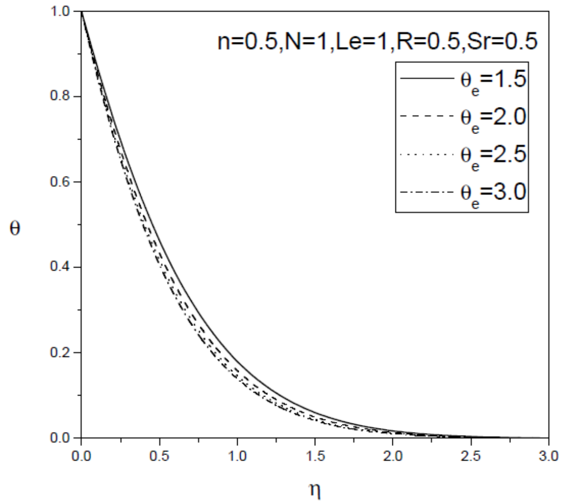

Figure 15.Temperature profile for various value of θ_e for pseudo-plastic fluid

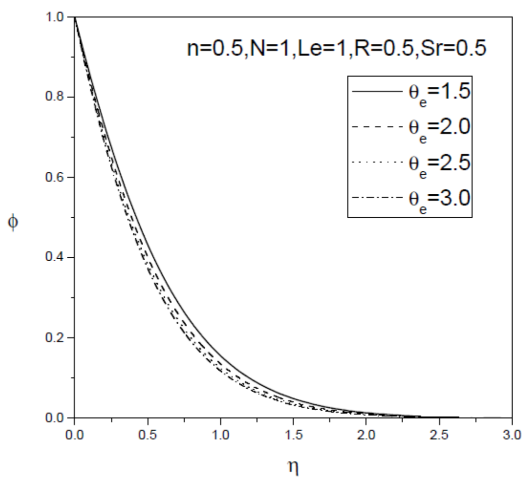

Figure 16.Concentration profile for various value of θ_e for pseudo-plastic fluid.

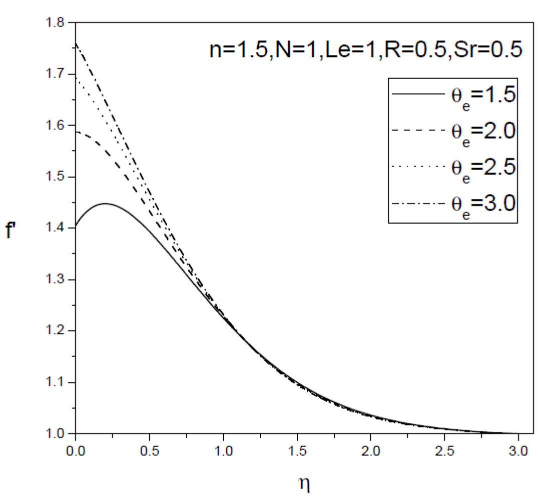

Figure 17. Velocity profile for various value of for θ_e dilatant fluids fluid.

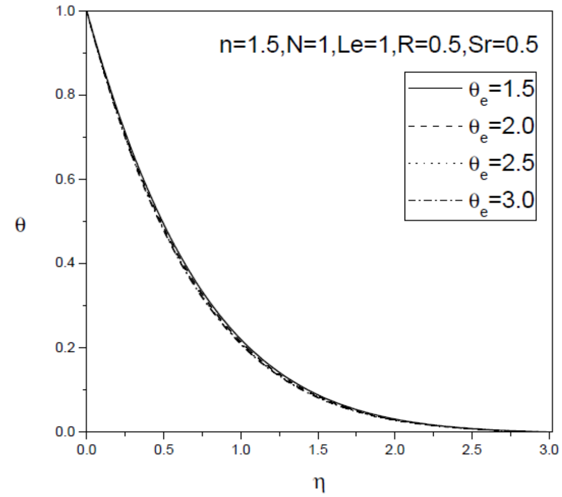

Figure 18. Temperture profile for various value of for θ_e dilatant fluids fluid.

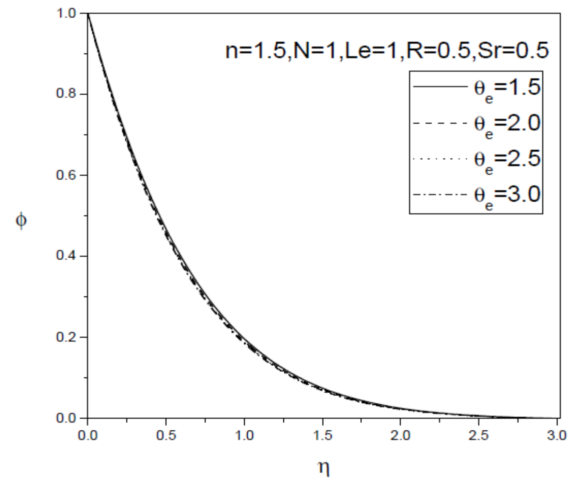

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Figure 19.Concentration profile for various value of for θ_e dilatant fluids fluid.

TABLE I

n	R	Sr	θ_e	Nu	Sh
0.5	0.5	0.5	2.0	2.721018	1.727446
1.0	*0.5*	*0.5*	*2.0*	2.519813	1.587523
1.5	*0.5*	*0.5*	*2.0*	2.578433	1.626200
0.5	*0.0*	*0.5*	*2.0*	2.020100	1.562625
0.5	*1.0*	*0.5*	*2.0*	3.401588	1.801532
0.5	*2.0*	*0.5*	*2.0*	4.746861	1.870592
0.5	0.5	0.0	2.0	2.698185	1.992165
0.5	*0.5*	*1.0*	*2.0*	2.744939	1.454161
0.5	*0.5*	*2.0*	*2.0*	2.796233	0.880436
0.5	0.5	0.5	1.5	2.542375	1.604364
0.5	0.5	0.5	2.5	2.833060	1.804602
0.5	0.5	0.5	3.0	2.909503	1.857226

III. CONCLUSION

In this study the effect of variable viscosity, radiation and Soret on mixed convection heat and mass transfer from a vertical plate in a Darcy power-law fluid saturated porous medium are investigated. It is noted that the velocity, temperature and concentration profiles are significantly effected by the variable viscosity, radiation and Soret parameter in the medium.

· Higher the values of radiation parameter results in lower velocity and concentration profiles in both pseudo-plastic and dilatant fluids but there is an increase in temperature profile for both fluids.

· An increase in the value of Soret parameter results in higher velocity and concentration profiles both pseudo-plastic and dilatant fluids where as slightly lower in temperature profile.

· Higher the values of variable viscosity results in higher velocity near the plate and lower velocity away from the plate for both pseudo-plastic fluid and dilatant fluid, but temperature and concentration profiles increase with decrease in the variable viscosity for both fluids.

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