An Integrated Production Model with Pollution Reduction Goals

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Abstract---An integrated linear programming model is generated in this paper by effectively incorporating production, logistics, and air pollution related analysis. This paper is suitable for certain firms that own multiple plants or factories producing multiple products. The objective is to find suitable production levels of each product at each plant and the optimal shipment quantities under environmental constraints with air pollution reduction aims.

Key words--- Linear programming, pollution reduction, production engineering

I. Introduction

THE methodology developed in this paper considers the previous optimization work[1] and the use of an index development procedure[2] ; the model design is based on an integrated model with index based objective function and with the environmental air pollution component added to the production and logistics components. The objective function of the linear programming model developed in this study consists of production return index, transportation cost index and air pollution index components. Each of these indices has been designed in such a way so that as we try to maximize the sum of these indices, the production monetary return index is maximized whereas transportation cost index and air pollution index are both minimized; this is because the transportation cost index and the air pollution index have been calculated in a reverse way as compared to production monetary return index. The index- based objective function is followed by a set of production, transportation, and environmental related constraints. Index computational procedure is explained in the next section prior to the model design.

II. Index Development Procedure

An innovative and unique indexing procedure is developed in order to combine entities that have different dimensions and measurement units as well as different objectives. For example, the return on products is monetary and measured in currency units where as air pollution is measured in amounts of certain particles emitted into the air. It is not possible to combine these different dimensions into a single common objective function unless an dimensionless indexing procedure that is free from specific measurement units is utilized. By using this special indexing procedure it is possible to combine different conflicting objectives such increasing monetary eeturns and decreasing air pollution into a single maximization objective function. The indexing methodology is :

1. Determine the highest r_{ij} and divide every r_{ij} by this highest r_{ij} to get the return index for each product *i* produced at plant or factory *j*. This is a case of direct proportionality because the higher the return per unit, the better is the total return. The Index on Returns would be denoted by IR_{ij} and would be given by $IR_{ij} = r_{ij} /Max\{ r_{ij}\}$
2. Determine the lowest c_{ijk} and divide this lowest c_{ijk} by every c_{ijk} to get the transportation Cost Index for each product *i* produced shipped from plant or factory *j* to demand center *k*. This is a case of *inverse* proportionality because the most desirable transportation route would be the one with the cheapest cost per unit and maximizing this transportation Cost Index would have the reverse effect of reducing total transportation cost. The transportation Cost Index denoted by IC_{ijk} would be given by $IC_{ijk} =Min\{ c_{ijk} \}/ c_{ijk}$
3. Determine the lowest a_{ij} and divide this lowest a_{ij} by every a_{ij} to get the Air pollution Index for each product *i* produced at plant or factory *j*. This is a case of *inverse* proportionality as the most desirable production source would be the one with least polluting factory and maximizing this air pollution index would have the reverse effect of reducing total air pollution. The Air pollution Index denoted by IA_{ij} would be given by $IA_{ij} =Min\{ a_{ij} \}/ a_{ij}$

The objective function of the linear program aims at maximizing the sum of:

$IR_{ij} = r_{ij} /Max\{ r_{ij}\}$,
$IC_{ijk} =Min\{ c_{ijk} \}/ c_{ijk}$
$IA_{ij} =Min\{ a_{ij} \}/ a_{ij}$.

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III. The Model Design

The objective function of following model is based on the indexing procedure and the model is designed accordingly. The objective is to determine suitable production levels to increase our total return while reducing transportation costs and amount of air pollution generated by the total production.

The objective function is followed by 3 constraint sets. Constraint Set(1) contains the environmental constraints on each production region. The emissions causing air pollution may be different at different plants or factories depending on the manufacturing process used in production of a certain product and the total permitted or tolerated amount of emitted pollution may vary from region to region as the amount of already existing air pollution may be different in each production region.

The Constraint Set(2) shows the relationship between the production and transportation variables. Plant or factories utilizing this model would produce to the extent that it is possible and profitable to transport and hence all the production would be shipped .

The Constraint Set(3) consists of demand constraints showing that amount of shipment of product *i* from all the plants or factories to market *k* is less than or equal to the demand on product *i* at market *k*; " less than or equal to" format is used because the demand will be met to the extent that is profitable rather than forcing the model to meet the exact demand at any cost.

The fully developed model is:

$$\text{Max} \sum_i \sum_j IR_{ij}\, x_{ij} + \sum_i \sum_j \sum_k IC_{ijk}\, x_{ijk} + \sum_i \sum_j IA_{ij}\, x_{ij}$$

st

$$\sum_i^m a_{ij}\, x_{ij} \leq e_j \qquad j=1,2,\ldots,n \qquad (1)$$

$$\sum_k^l x_{ijk} = x_{ij} \qquad i=1,2,\ldots,m \quad j=1,2,\ldots,n \qquad (2)$$

$$\sum_j^n x_{ijk} \leq d_{ik} \qquad i=1,2,\ldots,m \quad k=1,2,\ldots,l \qquad (3)$$

where

$IR_{ij} = r_{ij}/ \text{Max}\{r_{ij}\}$

$IC_{ijk} = \text{Min}\{c_{ijk}\}/ c_{ijk}$

$IA_{ij} = \text{Min}\{a_{ij}\}/ a_{ij}$

r_{ij} = unit return on product i at plant j

IR_{ij} = return index per unit of product i produced at plant j

c_{ijk} = unit cost of shipment for product i from plant j to market k

IC_{ijk} = transportation cost index per unit shipment of product *i* from plant *j* to market *k*

a_{ij} = amount of increase on the air pollution index per unit of product i produced at plant j

IA_{ij} = air pollution index per unit of product i produced at plant j

x_{ij} = amount of product i produced at plant j in a specific period

x_{ijk} = amount of shipment for product i from plant j to market k in a specific period

e_j =total amount of air pollution generated by plant j that is permitted or tolerated in region j

d_{ik} = demand for product i at market k in a specific period

IV. Conclusion

Production, distribution and air pollution reduction issues are combined into a single linear programming model through an original and unique approach. This model is expected to contribute to the fields of production engineering , logistics, and environmental engineering. Maximizing monetary returns should not be the only objective of the firms. Exercising social responsibility such as reduced air pollution should also be part of the goals of the firms. In addition to allocating more of the production resources to those factories that pollute the least, minimizing transportation cost index would generally mean reducing total distance traveled and hence contributing to less vehicle travel and thus less air pollution. This model takes into account those concerns in a comprehensive way.

References

[1] S Cokelez & J Burns, Distribution Systems-Warehouse Location and Capacity, *Omega, The International Journal of Management Science,*, Vol. 17, No. 1, pp. 45-51.

[2] S. Cokelez, An Optimization Methodology for Selecting Suppliers in Purchasing Management for Improved Customer Service, *Journal of Customer Service in Marketing & Management*, 3(3), 1997, pp. 7-17.