A Novel Spectral Relaxation Method for Mixed Convection in the Boundary Layers on an Exponentially Stretching Surface with MHD and Cross-Diffusion Effects

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***Abstract*—**This work is aimed to study of two-dimensional steady, mixed convection heat and mass transfer from an exponentially stretching vertical surface in an electrically conducting Newtonian fluid with specific forms of heat and mass fluxes by including the Soret and Dufour effects. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations, with the location along the plate as a parameter and then solved numerically using spectral relaxation method (SRM). A parametric study of the physical parameters involved in the problem is conducted, the results of this parametric study are shown graphically and the physical aspects of the problem are highlighted and discussed.

***Keywords*—**Soret and Dufour effects, MHD Mixed convection, Exponentially stretching surface, Spectral Relaxation Method (SRM).

I. Introduction

THE study of flow, heat and mass transfer in the boundary layer of a continuously stretching surface with a given temperature distribution moving in an otherwise quiescent fluid medium has attracted the attention of researchers for the past few decades due to its significant industrial and engineering applications. After the revolutionary works of Sakiadis[1], a number of researchers discussed the problem of boundary layer flow to obtain the thermal and kinematic behaviour by considering the different forms of stretching velocity (see Magyari and Keller [2]; Partha *etal.*[3] and Bidin & Nazar [4] and citation therein).

In double-diffusive (e.g., thermohaline) convection the coupling between temperature and solutal fields takes place because the density of the fluid mixture depends on both the temperature T and the concentration C (and also, in general, on the pressure P). In some circumstances there is direct coupling. This is when cross-diffusion (Soret and Dufour effects) is not negligible. Thermal diffusion, also called thermo-diffusion or Soret effect, corresponds to species differentiation developing in an initial homogeneous mixture subjected to a temperature gradient. The heat flux induced by a concentration gradient is called Dufour or diffusion-thermo effect. In the recent past, considerable attention has been paid to the theoretical and numerical study of these combined effects as it is considered as second order phenomena and are significant in areas such as hydrology, petrology and geosciences. Although the cross-diffusion effects of the medium on the convective transport in a viscous fluid with heat and mass fluxes are important, very little work has been reported in the literature. The Dufour effect was found to be of order of considerable magnitude such that it cannot be neglected (Eckert and Drake [5]). Dursunkaya and Worek [6] studied Soret and Dufour effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [7] presented the same effects on mixed convection on a vertical flat plate with temperature dependent viscosity. Due to its attractive applications in engineering and industrial application, many researchers considered mixed convection boundary layer flows in different fluid flow configurations in presence of cross-diffusion effects (eg., Abreu *et al.* [8]; Lakshmi Narayana & Murthy [9]; Rawat & Bhargava [10] and Srinivasacharya & RamReddy [11] etc.)

There has been a renewed interest in magneto-hydrodynamics (MHD) flow, heat and mass transfer in viscous and clear domains due to the important effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluid such as MHD power generators, the cooling of nuclear reactors, plasma studies, geothermal energy extractions etc. Accordingly, it is of interest to examine the effect of the magnetic field on the flow, heat and mass transfer characteristics. Studying such an effect has great importance in various application fields where MHD, Soret and Dufour effects are correlative. Hence, many problems of MHD, Soret and Dufour effects in viscous fluids by considering different surface geometries have been analyzed and reported in the literature. For example, Raptis & Singh [12]; Postelnicu [13]; Alam *etal*. [14] and Srinivasacharya *et al*. [15] etc studied the above said effects by considering Newtonian and non-Newtonian fluids along vertical plate.

To our best knowledge and from the literature, to date, there is no study which has considered this problem. The heat and mass fluxes and free stream flow are given specific forms of profiles which permit similarity solution. The spectral relaxation method is employed to solve the non-linear system

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in the present problem. The effects of magnetic, mixed convection parameters, Soret and Dufour numbers and also X-location are examined and some of them are displayed through graphs.

II. Mathematical Formulation

Consider a steady, two-dimensional and laminar flow of an incompressible and electrically conducting Newtonian fluid near an impermeable plane vertical wall stretching with velocity $u_w(x)$, heat flux $q_w(x)$ and mass flux $q_m(x)$ moving through a quiescent incompressible fluid of constant temperature T_∞ and concentration C_∞ as shown in the Fig.(1). The x-axis is directed along the continuous stretching surface and points in the direction of motion whereas the y-axis is perpendicular to plate and to the direction of slot (z-axis) whence the continuous stretching plane surface issues. A uniform magnetic field B_0 is assumed to be applied in the y-direction. It is assumed that the induced magnetic field of the flow is negligible in comparison with the applied one which corresponds to a very small magnetic Reynolds number. Further, u and v are the velocity components in the x- and y-directions. In addition, the Soret and Dufour effects are considered.

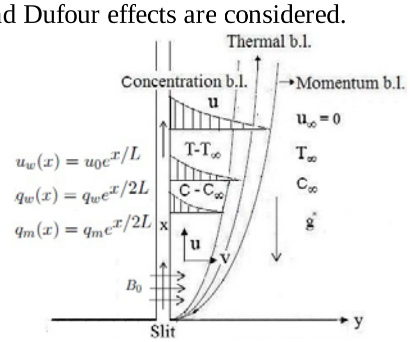


Fig. 1 Physical model and coordinate system

Using the above assumption, and the Boussinesq and boundary layer approximations, the governing equations for the electrically conducting Newtonian fluid are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}+g^*\beta_T\left((T-T_\infty)+\frac{\beta_C}{\beta_T}(C-C_\infty)\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{DK_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\frac{DK_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u and v are the velocity components in x and y directions respectively, T is the temperature, C is the concentration, g^* is the acceleration due to gravity, ρ is the density, B_0 is the strength of the magnetic field, σ is the electrical conductivity of the fluid, μ is the dynamic coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansions, α is the thermal diffusivity, D is the solutal diffusivity of the medium, C_p is the specific heat capacity, C_s is the concentration susceptibility, T_m is the mean fluid temperature and K_T is the thermal diffusion ratio. The last term on the right-hand side of the energy equation (3) and diffusion equation (4) signifies the diffusion-thermo effect and the thermal-diffusion effect respectively.

The boundary conditions are

$$u=u_w(x),\ v=0,\ -k\left(\frac{\partial T}{\partial y}\right)_w=q_w e^{x/L},\ -D\left(\frac{\partial C}{\partial y}\right)_w=q_m e^{x/L} \quad \text{at} \quad y=0 \tag{5a}$$

$$u=0, T=T_\infty,\ C=C_\infty \qquad as \qquad y\to\infty \tag{5b}$$

where the subscripts w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively, q_w is parameter of the temperature distribution where as q_m is parameter of the concentration distribution in the stretching surface. The stretching velocity $u_w(x)$ is defined as $u_w(x)=u_0 e^{x/L}$ where u_0 is velocity parameter of the stretching surface.

III. Solution Of The Problem

It is convenient to transform the governing equations into a dimensionless form which can be suitable for solution. This can be done by introducing the following dimensionless variables

$$\left.\begin{aligned}&\eta=\sqrt{\frac{\mathrm{Re}}{2}}\frac{y}{L}e^{x/2L},\ \psi=\sqrt{2}\,\nu\,\mathrm{Re}^{1/2}e^{x/2L}\ f(\eta)\\ &T(x,y)=T_\infty+\frac{\sqrt{2}q_w L}{k}Re^{-1/2}e^{x/2L}\theta(\eta),\\ &C(x,y)=C_\infty+\frac{\sqrt{2}q_m L}{D}Re^{-1/2}e^{x/2L}\phi(\eta)\end{aligned}\right\} \tag{6}$$

where ψ is the stream function that satisfies the continuity equation, which is defined as $u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}$. With the change of variables, Eq.(1) is identically satisfied and Eqs.(2)-(4) are transformed to:

$$f'''+ff''-2f'^2+2Rie^{-3X/2}\left(\theta+\mathrm{B}\phi\right)-2Me^{-X}f'=0 \tag{7}$$

$$\theta''+Pr\,(f\theta'-f'\theta+D_f\phi'')=0 \tag{8}$$

$$\phi''+Sc\,(f\phi'-f'\phi+S_r\theta'')=0 \tag{9}$$

where the primes indicate partial differentiation with respect to η alone, ν is the kinematic viscosity, L is the characteristic length of the plate, $X=(x/L)$ is the X-location, $Re=(u_0 L/\nu)$ is the Reynolds number, $Pr=(\nu/\alpha)$ is the Prandtl number, $Sc=(\nu/D)$ is the Schmidt number, $Ha^2=(\sigma B_0^2 L^2/\rho\nu)$ is the Hartman number, $M=(Ha^2/\mathrm{Re})$ is the magnetic parameter which is the ratio of Hartman number to Reynolds number, $Gr=(g^*\beta_T q_w L^4/k\nu^2)$ is the thermal Grashof number, $N=(\beta_C q_m k/\beta_T q_w D)$ is the buoyancy ratio, $D_f=(DK_T/C_s C_p\nu)(q_m k/q_m D)$ is the Dufour number and $S_r=(DK_T/T_m\nu)(q_w D/q_w k)$ is the Soret number and $Ri=(Gr/Re^{5/2})$ is the mixed convection parameter

Boundary conditions (5) in terms of f, θ and ϕ become

$$\eta=0:\ f=0,\ f'=1,\ \theta'=-1,\ \phi'=-1 \quad (10a)$$

$$\eta\to\infty:\ f'=0,\ \theta=0,\ \varphi=0 \quad (10b)$$

The non-dimensional skin friction $C_f=\left(2\tau_w/\rho U_*^2\right)$, the local Nusselt number $Nu_x=\left(x\,q_w(x)/k\left[T_w(x)-T_\infty\right]\right)$ and local Sherwood number $Sh_x=\left(x\,q_m(x)/D\left[C_w(x)-C_\infty\right]\right)$, where U* is the characteristic velocity, are given by

$$C_f\sqrt{Re_x}=\sqrt{2X}\,f''(0),\ \left(Nu_x/\sqrt{Re_x}\right)=\left(X\,e^{X/2}/\sqrt{2}\ \theta(0)\right),$$
$$\left(Sh_x/\sqrt{Re_x}\right)=\left(X\,e^{X/2}/\sqrt{2}\ \phi(0)\right) \quad (11)$$

where $\mathrm{Re}_x=\left(u_w(x)\,x/\nu\right)$ is the local Reynolds number based on the surface velocity.

IV. Numerical Procedure

The spectral relaxation method (SRM) is a recently proposed algorithm for the solution of boundary value problems of the form Eqs. (7) - (10). The algorithm uses the idea of the Gauss-Seidel method to decouple the governing systems of equations. From the decoupled equations an iteration scheme is developed by evaluating linear terms in the current iteration and non-linear terms in the next iteration level. The decoupled system of equations is solved using the Chebyshev pseudo - spectral method (For more details, see Canuto *et al.* [16] and Trefethen[17]], article by Kameswaran *et al.*[20] and the citations therein). The basic idea behind the spectral collocation method is the introduction of a differentiation matrix D which is used to approximate the derivatives of the unknown variables, for example, $f(\eta)$ at collocation points as the matrix vector product

$$\frac{df}{d\eta}=\sum_{k=0}^{\bar{N}}\mathbf{D}_{lk}\,f(\tau_k)=\mathbf{D}f,\ l=0,1,\ldots,\bar{N},$$

where $\bar{N}+1$ is the number of collocation points (grid points), $\mathbf{D}=2\mathrm{D}/\eta_\infty$, and $f=[f(\tau_0), f(\tau_1),\ldots, f(\tau_{\bar{N}})]^T$ is the vector function at the collocation points. Higher order derivatives are obtained as powers of $\mathbf{D}$, that is $f^{(p)}=\mathbf{D}^p\mathbf{Z}$, where p is the order of the derivatives, η_∞ is a finite length that is chosen to be numerically large enough to approximate the conditions at infinity in the governing problem and τ is a variable used to map the truncated interval $[0,\eta_\infty]$ to the interval $[-1,1]$ on which the spectral method can be implemented.

To apply the spectral relaxation method to (7) - (10), we first reduce the highest order derivative in equation (7) by setting $f'(\eta)=p(\eta)$. With the spectral relaxation method to Eqs. (7) - (10), we obtain the following iteration scheme:

$$p''_{r+1}+f_r\,p'_{r+1}+2\,Ri\,e^{-3X/2}(\theta_r+N\phi_r)-2M\,e^{-X}\,p_{r+1}=2\,p_r^2 \quad (12)$$

$$f'_{r+1}=p_{r+1},\ f_{r+1}(0)=0 \quad (13)$$

$$\theta''_{r+1}+\Pr(f_{r+1}\theta'_{r+1}-p_{r+1}\theta_{r+1}+D_f\,\phi''_r)=0 \quad (14)$$

$$\phi''_{r+1}+Sc(f_{r+1}\phi'_{r+1}-p_{r+1}\phi_{r+1}+S_r\,\theta''_{r+1})=0 \quad (15)$$

subject to boundary conditions:

$$p_{r+1}(0)=1,\ p_{r+1}(\infty)=0,\ \theta_{r+1}(0)=1,\theta_{r+1}(\infty)=0,$$
$$\phi_{r+1}(0)=1,\ \phi_{r+1}(\infty)=0 \quad (16)$$

Eqs. (12) - (15) are now written in the form:

$$A_1\,p_{r+1}=B_1,\ p_{r+1}(\tau_{\bar{N}})=1,\ p_{r+1}(\tau_0)=0 \quad (17)$$

$$A_2\,f_{r+1}=B_2,\ f_{r+1}(\tau_{\bar{N}})=0 \quad (18)$$

$$A_3\theta_{r+1}=B_3,\sum_{k=0}^{N}D_{Nk}\theta_{r+1}(\tau_k)=-1,\ \theta_{r+1}(\tau_0)=0 \quad (19)$$

$$A_4\phi_{r+1}=B_4,\sum_{k=0}^{N}D_{Nk}\phi_{r+1}(\tau_k)=-1,\ \phi_{r+1}(\tau_0)=0 \quad (20)$$

where

$$A_1=\mathrm{D}^2+\mathrm{diag}[f_r]\mathrm{D}-2M\,e^{-X}\,\mathrm{I}$$
$$+2\,Ri\,e^{-3X/2}(\mathrm{diag}[\theta_r]+N\,\mathrm{diag}[\phi_r]),\ B_1=2\,p_r^2 \quad (21)$$

$$A_2=\mathrm{D},\ B_2=p_{r+1} \quad (22)$$

$$A_3=\mathrm{D}^2+\Pr(\mathrm{diag}[f_{r+1}]\,\mathrm{D}-\mathrm{diag}[p_{r+1}]\,\mathrm{I}),\ B_3=-\Pr D_f\ \phi''_r \quad (23)$$

$$A_4=\mathrm{D}^2+Sc(\mathrm{diag}[f_{r+1}]\,\mathrm{D}-\mathrm{diag}[p_{r+1}]\,\mathrm{I}),\ B_4=-Sc\ S_r\ \theta''_{r+1} \quad (24)$$

In Eqs. (21) - (24), **I** is an identity matrix, **D** is the differentiation matrix. The size of the matrix **O** is $(\bar{N}+1)\times 1$ and diag[ ] is a diagonal matrix, all of size $(\bar{N}+1)\times(\bar{N}+1)$, where $\bar{N}$ is the number of grid points, f, p, θ and ϕ are the values of the functions f, p, θ and ϕ when evaluated at the grid points. The subscript r denotes the iteration number.

The initial guesses to start the SRM scheme (12)- (15) are:

$$f_0(\eta)=1-e^{-\eta},\ p_0=e^{-\eta},\ \theta_0=e^{-\eta},\ \phi_0=e^{-\eta},$$

which are chosen to satisfy all the boundary conditions.

V. Results And Discussions

A new SRM has been used to analyze the combined Soret and Dufour effects on mixed convection flow, heat and mass transfer from an exponentially stretching vertical surface in an electrically conducting Newtonian fluid. We established the accuracy of the SRM by comparing the SRM results with those obtained using the Keller-box method. The results of these comparisons are shown in Table (1). The results from the two methods are in excellent agreement with the spectral relaxation method (SRM) converging at the fourth order with accuracy of up to six decimal places. This lends confidence to the numerical results reported below. The values $Pr = 1.0$, $Sc = 0.22$ and $B = 0.5$ are fixed unless otherwise mentioned.

The effects of the M on the dimensionless velocity, temperature and concentration are depicted for fixed values of Ri, S_r, D_f and *X*-location in the Figs. (1)-(3). It can be seen from Fig. (1) that the velocity reduces as M enhances. Fig. (2)-(3) indicate that a rise in M enhances the temperature and concentration in the Newtonian fluid.

Fig.(4) displays the non-dimensional velocity for different values of Soret number S_r and Dufour number D_f with $Ri = 1.0, X = 0.5, M = 0.5$. It is observed that the velocity of the fluid decreases with smaller values D_f and increases with larger values of D_f (or a decrease of S_r). The dimensionless temperature for different values of S_r and D_f for $Ri = 1.0, X = 0.5, M = 0.5$, is shown in Fig.(5). It is clear that the temperature of the fluid increases with the increase of the D_f (or decrease S_r). Fig.(6) demonstrates the dimensionless concentration for different values of S_r and D_f for $Ri = 1.0, X = 0.5, M = 0.5$. It is seen that the concentration of the fluid decreases with increase of the D_f (or a decrease of S_r).

The variations of the skin friction, local heat transfer and the local mass transfer coefficients for various values of the mixed convection parameter *Ri* with *M=0.5*, $X = 0.5$, $S_r = 2.0, D_f = 0.03,$, are shown in Table (2). It is seen that the local skin friction factor, the heat and mass transfer rates increase with the increasing value of *Ri*. Table (2) depict the variations of the skin friction coefficient $f''(0)$, local heat transfer coefficient $(1/\theta(0))$ and the local mass transfer coefficient $(1/\phi(0))$ for various values of the magnetic number M with $Ri = 5.0, S_r = 2.0, D_f = 0.03, X = 0.5$. It is observed that the skin friction, local heat transfer and the local mass transfer decrease with increasing values of the magnetic number *M*. Again, the skin friction, local heat and mass transfer coefficients are predicted to increase as the *X*-location increases. Table (2) illustrate the variations of the skin friction, local heat transfer and the local mass transfer coefficients for various values of the Soret number S_r and Dufour number D_f with $M = 0.5, Ri = 5.0, X = 0.5$. It is to be noted from Table (2) that simultaneously increasing D_f and decreasing S_r lead to initial decreases in the skin-friction coefficient $f''(0)$ up to $S_r = 0.12$, $D_f = 0.5$ and then start increasing. The heat transfer coefficient shows a monotonic decrease, while the mass transfer coefficient exhibits the opposite change when subjected to simultaneous increase in D_f and decrease in S_r. This is due to the coupling between the momentum, energy, and species balance equations, the Dufour parameter has an effect on the concentration boundary layer as well.

VI. Conclusions

The problem of mixed convection heat and mass transfer in an electrically conducting Newtonian fluid over an exponentially stretching surface in the presence of cross-diffusion effects is considered. The numerical results indicate that the skin friction coefficient as well as rate of heat and mass transfers in the MHD Newtonian fluid with magnetic field are lower compared to those of the Newtonian fluid without magnetic field. It is also seen that simultaneously increasing Dufour number and decreasing Soret number lead to initial decreases in the skin-friction coefficient up to a critical value and then start increasing. It is observed that the heat transfer coefficient monotonically decreased, while the mass transfer coefficient exhibited the opposite change when subjected to simultaneous increase in Dufour number and decrease in Soret number.

TABLE I

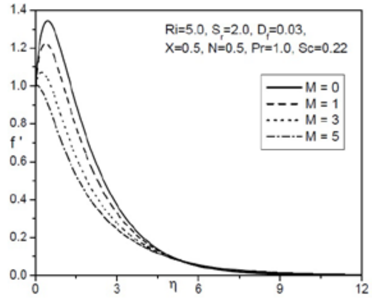
COMPARISON BETWEEN SKIN FRICTION COEFFICIENT $f''(0)$ CALCULATED BY THE PRESENT METHOD AND THAT OF RESULTS OBTAINED USING THE KELLER-BOX METHOD FOR $S_r = 2,\ D_f = 0.03,\ X = 5$ AND $M = 0.5$.

Ri	*Keller-box method*	*Present*
-0.5	-1.28580	-1.28588
-0.1	-1.28472	-1.28477
0.5	-1.28312	-1.28313
3.0	-1.27670	-1.27671
5.0	-1.27186	-1.27188

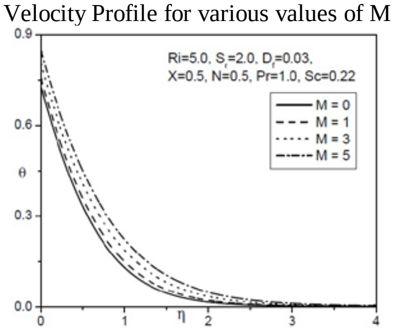
TABLE II

EFFECTS OF SKIN FRICTION, HEAT AND MASS TRANSFER COEFFICIENTS FOR VARYING VALUES OF Ri, S_r, D_f, X and M

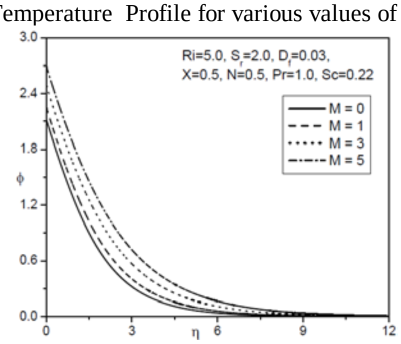
Ri	S_r	D_f	X	M	$f''(0)$	$\frac{1}{\theta(0)}$	$\frac{1}{\phi(0)}$
-.5	2.0	0.03	5.0	0.5	-1.28588	0.93903	0.23390
-0.1	2.0	0.03	5.0	0.5	-1.28477	0.93984	0.23477
0.5	2.0	0.03	5.0	0.5	-1.28313	0.94100	0.23599
3.0	2.0	0.03	5.0	0.5	-1.27671	0.94518	0.24023
5.0	2.0	0.03	5.0	0.5	-1.27188	0.94803	0.24300
1.0	2.0	0.03	0.5	0.5	-0.53753	1.11854	0.34934
1.0	1.2	0.05	0.5	0.5	-0.58869	1.09985	0.38165
1.0	1.0	0.06	0.5	0.5	-0.60105	1.09302	0.39092
1.0	0.5	0.12	0.5	0.5	-0.62750	1.06200	0.41726
1.0	0.2	0.3	0.5	0.5	-0.62330	0.99261	0.43820
1.0	0.1	0.6	0.5	0.5	-0.58869	0.90120	0.45057
1.0	2.0	0.03	0.1	0.5	-0.09620	1.17603	0.37676
1.0	2.0	0.03	0.5	0.5	-0.53753	1.11854	0.34934
1.0	2.0	0.03	1.0	0.5	-0.87651	1.06237	0.32052
1.0	2.0	0.03	2.0	0.5	-1.17274	0.99194	0.27878
5.0	2.0	0.03	0.5	0.0	1.83496	1.37049	0.47057
5.0	2.0	0.03	0.5	1.0	1.39904	1.32190	0.44343
5.0	2.0	0.03	0.5	3.0	0.68946	1.24175	0.40139
5.0	2.0	0.03	0.5	5.0	0.11695	1.17784	0.37051



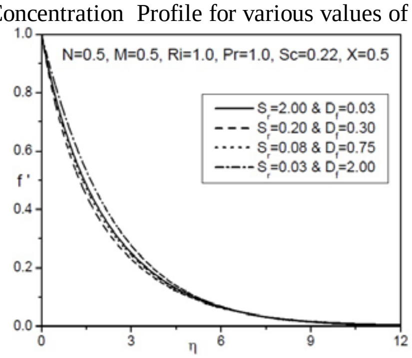
Velocity Profile for various values of M



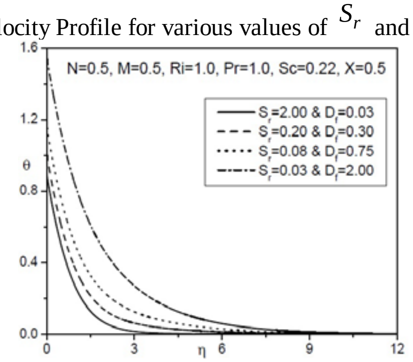
Temperature Profile for various values of M



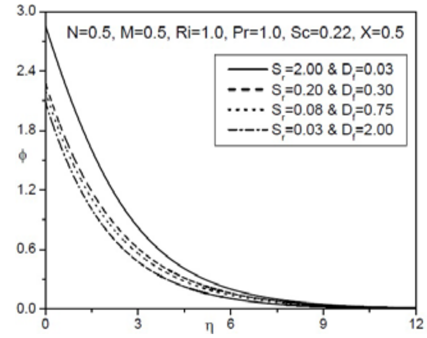
Concentration Profile for various values of M



Velocity Profile for various values of S_r and D_f



Temperature Profile for various values of S_r and D_f .



Concentration Profile for various values of S_r and D_f .

REFERENCES

[1] B.C. Sakiadis, ``Boundary-layer behavior on continuous solid surfaces''. *AIChE J*. vol. 7, pp.26-28, 1961.

[2] E. Magyari and B. Keller, ``Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface''. *J. Phys. D: Appl. Phy*. vol.32, pp.577-585, 1999.

[3] M.K. Partha, P.V.S.N. Murthy and G.P. Rajasekhar, ``Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface''. *Heat Mass Transfer*, vol. 41, pp.360-366, 2005.

[4] B. Bidin and R. Nazar, ``Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation''. *European Journal of Scientific Research*, vol.33, pp.710-717, 2009.

[5] E.R.G. Eckeret and R.M. Drake, ``*Analysis of heat and mass transfer''*. McGraw Hill, Newyark, 1972.

[6] Z. Dursunkaya and W.M. Worek, ``Diffusion-thermo and thermal diffusion effects in transient and steady natural convection from a vertical surface''. *Int.J.Heat Mass Transfer,* vol.35, pp.2060-2065, 1992.

[7] N.G. Kafoussias and N.G. Williams, ``Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity''. *Int. J. Engng. Sci.* vol. 33, pp.1369-1384, 1995.

[8] C.R.A. Abreu, M.F. Alfradique and A.T. Silva, ``Boundary layer flows with Dufour and Soret effects: I: Forced and natural convection''. *Chemical Engineering Science,* vol. 61, pp.4282-4289, 2006.

[9] P.A. Lakshmi Narayana and P.V.S.N. Murthy, ``Soret and Dufour effects in a doubly stratified Darcy porous medium''. *Journal of Porous Media*, vol.10, pp.613-624, 2007.

[10] S. Rawat and R. Bhargava, ``Finite element study of natural convection heat and mass transfer in a micropolar fluid saturated porous regime with Soret/Dufour effects''. *Int.J.of Appl. Math Mech*.5, pp.58-71, 2009.

[11] D.Srinivasacharya and Ch.RamReddy, ``Effects of Soret and Dufour on Mixed Convection Heat and Mass Transfer in a Micropolar Fluid with Heat and Mass Fluxes''. *Int. J. of Appl. Math Mech*. 6, pp.8-21, 2010.

[12] A. Raptis and A.K. Singh, ``MHD free convection flow past an accelerated vertical plate''. *Int. Commun. Heat Mass Transfer*. vol. 10, pp.313-321, 1983.

[13] A. Postelnicu, ``Influence of a magnetic field on heat and mass transfer by natural convection from vertical sufaces in porous media considering Soret and Dufour effects''. *Int. J. of Heat Mass Transfer* 47, pp.1467-1475, 2004.

[14] S. Alam, M.M. Rahman, A. Maleque and M. Ferdows, ``Dufour and Soret effects on steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate''. *Thammasat Int. J. Tech*. vol. 11, pp.1-12, 2006.

[15] D. Srinivasacharya D, J. Pranitha and Ch. RamReddy, ``Magnetic Effect on Non-Darcy Free Convection in a Power-Law Fluid with Double Stratification''. *J. of the Braz. Soc. of Mech. Sci. and Eng*. vol.33, pp.8-14, 2011.

[16] C. Canuto, M.V. Hussaini, A. Quarteroni and T.A**.** Zang, *Spectral Methods in Fluid Dynamics.* Springer – Verlag, 1958.

[17] L.N. Trefethen, *Spectral Methods in MATLAB*. SIAM, 2000.

[18] Kameswaran PK, Makukula ZG, Sibanda P, Motsa SS and Murthy PVSN (2013). A new algorithm for internal heat generation in nanofluid flow due to a stretching sheet in a porous medium. *Int. J. of Numerical Methods for Heat & Fluid Flow* 24(5), pp.1020-1043.