An Integrated Inventory Model with Retail Price-dependent Demand in Response to Announced Supply Price Increase

Chih-Te Yang, Liang-Yuh Ouyang, Yen-Wen Wang, and Chun-Ming Yang

Abstract**—**This paper proposes a single-vendor, single-buyer integrated inventory model in response to announced supply price increase when market demand rate depends on the retail price. The purpose of this paper is to determine the optimal special ordering and pricing policies for the buyer and the optimal number of shipment per production cycle for the vendor. Furthermore, when the buyer makes a special order, the vendor may or not provide all the special order quantity at the buyer's next replenishment date and hence the shortage may or not occur. That is, two specific situations are discussed in this study. A simple algorithm to find the optimal solution is developed. Finally, an numerical example will be presented to demonstrate the integrated model and solution procedure and provide manager a useful decision consultation.

Keywords**—**Inventory, integrated model, supply price increase, retail price-dependent demand.

I. Introduction

SINCE Naddor [1] presented an economic order quantity (EOQ) model where the vendor announces a supply price increase, many researchers have taken the price increase into account and proposed various analytical models to gain more insight into the relationship between price increase and inventory policy (see for example, [2]–[9]). When taking the price increase into account, it is common that in response to the price increase, the buyer may have no choice but to increase the retail price. Nevertheless, a weakness in above inventory models is they neglect the price-dependent demand rate, a common phenomenon.

In real business environment, pricing strategy is one of the major policies for the buyer to obtain its maximum profit. Usually, price has a direct impact on demand, and models with price-dependent demand occupy a prominent place in the inventory literature [10]. Cohen [11] firstly determined the optimal replenishment cycle and price for inventory that is subject to continuous decay over time at a constant rate. In after years, There is a vast amount of literature on inventory models for price-dependent demand rate such as Wee [12], Abad [13], Wee and Law [14], Abad [15], Mukhopadhyay et al. [16]–[17], Chang et al. [18], Dye [19] etc.

In addition, the objective of effective supply chain management is the reduction of costs, improvement of cash flow and increased operational efficiency across the entire business through connecting inventory control, purchasing coordination and sales order processing with market demand [20]. The joint optimization concept for the vendor and buyer is initiated by Goyal [21]. Banerjee [22] extended Goyal's [21] model and assumed that the supplier followed a lot-for-lot shipment policy with respect to a retailer. Goyal [23] extended Banerjee's model by relaxing the lot-for-lot assumption and assumed that the vendor's lot size is an integer multiple of the buyer's order size and examined a model for a single vendor-single buyer production inventory system. Lu [24] then generalized Goyal's [23] model by relaxing the assumption that the supplier could supply the retailer only after completing the entire lot size. Following, Many researchers (see for example, [25]–[30]) continued to propose more batching and shipping policies for integrated inventory models.

Consequently, the contribution of this paper, relative to previous studies, is that we explore a vendor-buyer integrated inventory in the context of the following three issues: (1) when the buyer is informed by the vendor of a future price increase and decides whether to make a special order before the increase, and what their new retail price should be; and (2) the vendor's lot size is an integer multiple of the buyer's order size. Furthermore, due to the vendor may or not provide all the special order quantity at the buyer's next replenishment date and hence the shortage will or not occur, two specific situations are discussed in this study.

II. Notation and Assumptions

The following notation and assumptions are used in this study:

A. Notation

c vendor's unit production cost before the material price increase

c_r vendor's unit production cost after the material price increase, c_r>c

Chih-Te Yang is with the Department of Industrial Management, Chien Hsin University of Science and Technology, Jung-Li, Taoyuan, Taiwan (e-mail:ctyang@uch.edu.tw).

Liang-Yuh Ouyang is with the Department of Management Sciences, Tamkang University, Taipei, Taiwan (e-mail:liangyuh@mail.tku.edu).

Yen-Wen Wang is with the Department of Marketing and Distribution Management, Chien Hsin University of Science and Technology, Jung-Li, Taoyuan, Taiwan (corresponding author's phone:886-3-4581196#6120; fax: 886-3-2503908; e-mail: ywwang@uch.edu.tw)

Chun-Ming Yang is a Ph.D. student of the Department of Management Sciences, Tamkang University, Taipei, Taiwan (e-mail: vicyang0706@ gmail.com).

v vendor's unit supply price (i.e., buyer's unit purchasing cost) before the material price increase, $v>c$

v_r vendor's unit supply price (i.e., buyer's unit purchasing cost) after the material price increase, $v_r>c_r$

p buyer's unit retail price when the unit purchasing cost is v, $p>v$

p_r buyer's unit retail price when the unit purchasing cost is v_r, $p_r>v_r$

p_s buyer's unit retail price for the special order quantity, a decision variable

$D(p)$ market demand rate, which is a decreasing function of the unit retail price.

S vendor's setup cost per setup

A buyer's ordering cost per order

R vendor's production rate

h_v vendor's holding cost rate, as a fraction of the cost of the item carried in inventory per unit time, $0<h_v<1$

h_b buyer's holding cost rate, as a fraction of the cost of the item carried in inventory per unit time, $0<h_b<1$

π buyer's unit shortage cost per unit time

Q buyer's order quantity before the material price increase

Q_r buyer's order quantity after the material price increase

Q_s buyer's special order quantity (i.e., vendor's special production quantity) before the material price increase, a decision variable

m the number of shipment from the vendor to the buyer per production cycle before the material price increase

m_r the number of shipment from the vendor to the buyer per production cycle after the material price increase

m_s the number of shipment per production cycle when the retailer make a special order, an integer decision variable

T the length of buyer's replenishment cycle time before the material price increase

T_r the length of buyer's replenishment cycle time after the material price increase

T_s depletion time for the quantity Q_s, a decision variable

t_s the length of time in which the inventory is shortage, a decision variable

$JTP(p,T)$ joint total profit per unit time during the replenishment period T

$JTP_r(p_r,T_r)$ joint total profit per unit time during the replenishment period T_r

$g_1(m_s, p_s, T_s)$ joint total profit increase between the special order and regular order during the special cycle time for case 1

$g_2(m_s, p_s, t_s, T_s)$ joint total profit increase between the special order and regular order during the special cycle time for case 2

$*$ The superscript represents optimal value.

B. Assumptions

1. There is single-vendor and single-buyer for a single product in this model.
2. Before the material price increase, the buyer orders optimal economic order quantity, Q^* units per order; the vendor produces $m^* Q^*$ units in each production run, and delivers Q^* units to the buyer in each shipment. After the material price increase, if the buyer makes a special order with Q_s units, the vendor produces $Q_s+(m_s-1)Q_r^*$ units in a special production run. Otherwise, if the buyer makes a regular order with Q^* units, the vendor produces $Q^*+(m_r^*-1)Q_r^*$ units in a regular production run.
3. For the vendor, shortages are not allowed whether the buyer makes a general or special order. For the buyer, shortages are allowed when making a special order.
4. When shortages occur, the unsatisfied demand is complete backlogged.
5. When the material price increases, the vendor will reflect it on the supply price (i.e., the buyer's purchase cost). In turn, the buyer will also reflect its purchase cost on the retail price.
6. The demand rate is a non-negative, decreasing and concave function of the retail price p.

III. Model formulation

This study explores the possible effects of retail price increases on a retailer's replenishment policy. Before the material price increase, the integrated inventory system evolves as follows: the buyer orders Q units per order and the vendor produces mQ units in each production run, and delivers Q units to the buyer in each shipment. After the material price increase, the retailer determines whether make a special order or regular order. If the buyer decides to make a special order with Q_s units, the vendor produces $Q_s+(m_s-1)Q_r^*$ units in a special production run. Otherwise, if the buyer makes a regular order with Q^* units, the vendor produces $Q^*+(m_r^*-1)Q_r^*$ units in a regular production run. Following, we first establish the joint total profit per unit time before the price of material increases. And then the joint profit increase between the special order and regular order during the special cycle time is developed.

Before the price of material increases, the buyer's total profit per unit time consists of the selling revenue, ordering cost, purchasing cost and holding cost, which is given by

$$TPB(p, T) = (p-v)D(p)-A/T-[h_b vD(p)T]/2. \tag{1}$$

The vendor's total profit per unit time consists of the selling revenue, setup cost, production cost and holding cost, which is given by

$$TPV(m, p, T) = (v-c)D(p)-S/(mT) -\{[h_v cD(p)T]/2\}\times\{m-1+2-[mD(p)]/R\}. \tag{2}$$

Once the buyer and vendor have built up a long-term strategic partnership, they can jointly determine the optimal policy for both parties. Accordingly, the joint total profit per unit time can obtained as the sum of the buyer's and the vendor's total profits per unit time. That is,

$$JTP(m, p, T) = TPB(p, T)+TPV(m, p, T), \quad (3)$$

where $TPB(p, T)$ and $TVB(m, p, T)$ are shown in (1) and (2).

The objective of this problem is to determine the optimal pricing, ordering and production policies that correspond to maximizing the joint total profit per unit time. The optimal solutions can be obtained by using the following search procedure: Firstly, for fixed p and T, checking the effect of m on the joint total profit per unit time $JTP(m, p, T)$ in (3). Taking second-order derivative of $JTP(m, p, T)$ with respect to m, it gets $d^2JTP(m, p, T)/dm^2=-2S/m^3T<0$. Hence, $JTP(m, p, T)$is a concave function of m. Consequently, the search for the optimal number of shipments m (denoted by m^*) is reduced to find a local maximum.

Next, for a given integer m, we can prove that for any given retail price p, the optimal value of T not only exists but also is unique. And then for any given value of T, there exists a unique sell pricing p to maximize the objective function. The processes of proofs are similar to Dye [19], Wu et al. [10], Yang et al. [31], and hence are omitted here. Once the optimal retail price, p^*, the length of replenishment cycle time, T^*, and the number of shipment for the vendor to the buyer per production cycle, m^*, are calculated, the optimal order quantity, Q^*, the joint total profit per unit time, $JTP(m^*, p^*, T^*)$, can be obtained.

When the supply price changes from v to v_r due to the production cost changes from c to c_r, if the buyer does not replace a special order before the price increases, then he/she will reflect the supply price changes on the retail price. Hence, the retail price increases from p to p_r. In this situation, the joint total profit per unit time becomes

$$\begin{aligned}JTP_r(m_r, p_r, T_r) &= TPB_r(p_r, T_r)+TPV_r(m_r, p_r, T_r)\\ &= (p_r-c_r)D(p_r)-A/T_r-h_bv_rD(p_r)T_r/2-S/m_rT_r\\ &\quad -[h_vc_rD(p_r)T_r/2]\{m_r-1+[2-m_rD(p_r)]/R\}, \quad (4)\end{aligned}$$

where

$$TPB_r(p_r, T_r) = (p_r -v_r)D(p_r)-A/T_r -[h_bv_rD(p_r)T_r]/2,$$

and

$$\begin{aligned}TPV_r(m_r, p_r, T_r) &= (p_r-c_r)D(p_r)-S/m_rT_r\\ &\quad -\{[h_vc_rD(p_r)T_r]/2\}\{m_r-1+[2-m_rD(p_r)]/R\}.\end{aligned}$$

By using the similar argument as above, once the optimal retail price, p_r^*, the length of replenishment cycle time, T_r^*, and the number of shipment from the vendor to the buyer per production cycle, m_r^*, are calculated, the optimal order quantity, Q_r^*, and the joint total profit per unit time, $JTP(m_r^*, p_r^*, T_r^*)$, can be obtained.

Subsequently, when the vendor announces a supply price increase (from v to v_r) that is effective starting at the next production cycle, the buyer may at once place a special order to take advantage of the relative lower supply price before the price increases. In order to response the marketing situation, the buyer will reflect supply price changes on retail price.

Our purpose is to determine the optimal special order quantity and the retail price by maximizing the joint total profit increase between special and regular orders during the depletion time of the special order quantity. Due to the vendor may or not provide all the special order quantity at the buyer's next replenishment date and hence the shortage will or not occur, two specific situations are discussed in this study: (i) $Q_s/R \leqq T^*$ and (ii) $Q_s/R>T^*$. Next, we will formulate the corresponding joint total profit increasing function for these two cases.

Case 1. $Q_s/R \leqq T^*$

In this case, the vendor can provide all the special order quantity at the buyer's next replenishment date which implies the shortage will not occur (see Fig. 1). For the buyer, the total profit of the special order during the time interval [0, T_s] is equal to total revenue minus the total relevant cost which consists of the ordering cost, purchasing cost and holding cost, and can be expressed by $(p_s-v)D(p_s)T_s-A-h_bvD(p_s)T_s^2/2$. As to the following period, the buyer follows regular EOQ policies with the unit production cost c_r, purchasing price v_r and retail price p_r. Thus, the total profit during the rest period is $(m_s-1)T_r^* \times TPB_r(p_r^*, T_r^*)$.

Therefore, the total profit for the buyer in a special production cycle (denoted by $TPBS_1(m_s, p_s, T_s)$) is give by

$$\begin{aligned}TPBS_1(m_s, p_s, T_s) &= (p_s-v)D(p_s)T_s-A-[h_bvD(p_s)T_s^{2]}/2\\ &\quad +(m_s-1)T_r^* \times TPB_r(p_r^*, T_r^*). \quad (5)\end{aligned}$$

Similarly, the total profit for the vendor in a special production cycle (denoted by $TPVS_1(m_s, p_s, T_s)$) is equal to total revenue minus the total relevant cost which consists of the set-up cost, production cost and holding cost, and can be expressed by

$$\begin{aligned}TPVS_1(m_s, p_s, T_s) &= (v-c_r)D(p_s)T_s+(v_r-c_r)(m_s-1)D(p_r^*)T_r^*-S\\ &\quad -(h_vc_r/2)\{\{[D(p_s)T_s]^2-[(m_s-1)D(p_r^*)T_r^*]^2\}/R\\ &\quad +2(m_s-1)D(p_r^*)T_r^*T_s+(m_s-1)(m_s-2)D(p_r^*)T_r^{*2}\}. \quad (6)\end{aligned}$$

Consequently, the joint total profit for a special production cycle when the buyer decides to adopt a special order policy (denoted by $JTPS_1(m_s, p_s, T_s)$) can be obtained as the sum of the buyer's and the vendor's total profits, i.e.,

$$JTPS_1(m_s, p_s, T_s) = TPBS_1(m_s, p_s, T_s)+TPVS_1(m_s, p_s, T_s). \quad (7)$$

If the vendor and buyer adopt its regular policies, then the joint total profit for a special production cycle will be divided into two periods (see Fig. 1). For the buyer, the buyer orders Q^* units with the unit purchasing price v and sells with the retail price p^* in first replenishment cycle. At the following period, the buyer orders Q_r^* units at the unit purchasing price v_r and sells with the retail price p_r^*. The total profit for a special production cycle is given by

$$\begin{aligned}TPBN_1(m_s, T_s) &= T^*\, TPB(p^*, T^*)\\ &\quad +[T_s+(m_s-1)T_r^*-T^*]\, TPB(p_r^*, T_r^*). \quad (8)\end{aligned}$$

As to the vendor, in first regular production cycle, he/she products $Q^*+(m_r^*-1)Q_r^*$ units and then $m_r^*Q_r^*$ for the following production cycle. The total profit for the vendor in a special production cycle is

$$\begin{aligned}TPVN_1(m_s, T_s) &= (v-c_r)D(p^*)T^*+(v_r-c_r)(m_r^*-1)D(p_r^*)T_r^*-S\\ &\quad -(h_vc_r/2)\{\{[D(p^*)T^*]^2-[(m_r^*-1)D(p_r^*)T_r^*]^2\}/R\\ &\quad +2(m_r^*-1)D(p_r^*)T_r^*T^*+(m_s-1)(m_s-2)D(p_r^*)T_r^{*2}\}\\ &\quad +[T_s+(m_s-m_r^*)T_r^*-T^*]/[D(p_r^*)/R+(m_r^*-1)T_r^*]\\ &\quad \times m_r^*T_r^*TPV_r(m_r^*, p_r^*, T_r^*). \quad (9)\end{aligned}$$

Consequently, the joint total profit for a special production cycle when the buyer decides to adopt a regular order policy (denoted by $JTPN_1(m_s, T_s)$)can be obtained as the sum of the buyer's and the vendor's total profits, which leads to

$$JTPN_1(m_s\ T_s) = TPBN_1(m_s, T_s) + TPVN_1(m_s, T_s). \qquad (10)$$

Comparing (7) with (10), the joint total profit increase for Case 1 can be given by

$$g_1(m_s,\ p_s, T_s) = JTPS_1(m_s,\ p_s, T_s) - JTPN_1(m_s, T_s). \qquad (11)$$

where $JTPS_1(m_s, p_s, T_s)$ and $JTPN_1(m_s, T_s)$ are shown as in (7) and (10), respectively.

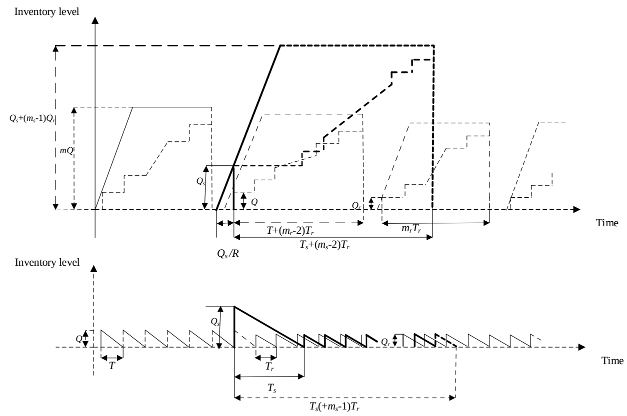

Fig. 1. The integrated inventory system when $Q_s/R \leqq T^*$

Case 2. $Q_s/R>T^*$

In this case, the vendor is unable to provide all the special order quantity at the buyer's next replenishment date and hence the shortage will occur and complete backlogged (see Fig. 2). For the buyer, the total profit of the special order during the time interval $[0, T_s]$ is equal to total revenue minus the total relevant cost which consists of the ordering cost, purchasing cost, holding cost and shortage cost and can be expressed by $(p_s\text{-}v)D(p_s)T_s\text{-}A\text{-}[h_bvD(p_s)(T_s\text{-}t_s)^2]/2\text{-}[\pi D(p_s)t_s^2]/2$.

As to the following period, the buyer follows regular EOQ policies with the unit production cost c_r, purchasing price v_r and retail price p_r. Thus, the total profit during the rest period is $(m_s\text{-}1)T_r^*TPB(p_r^*, T_r^*)$.

Therefore, the total profit for the buyer in a special production cycle (denoted by $TPBS_2(p_s, t_s, T_s)$) is give by

$$TPBS_2(p_s, t_s, T_s) = (p_s\text{-}v)D(p_s)T_s\text{-}A\text{-}[h_bvD(p_s)(T_s\text{-}t_s)^2]/2$$
$$\text{-}[\pi D(p_s)t_s^2]/2+(m_s\text{-}1)T_r^*TPB_r(p_r^*, T_r^*). \qquad (12)$$

Similarly, the total profit for the vendor in a special production cycle (denoted by $TPVS_2(m_s, p_s, t_s, T_s)$) is equal to total revenue minus the total relevant cost which consists of the set-up cost, production cost and holding cost, and can be expressed by

$$TPVS_2(m_s, p_s, t_s, T_s) = (v\text{-}c_r)D(p_s)T_s+(v_r\text{-}c_r)(m_s\text{-}1)D(p_r^*)T_r^*\text{-}S$$
$$\text{-}(h_vc_r/2)\{\{[D(p_s)T_s]^2\text{-}[(m_s\text{-}1)D(p_r^*)T_r^*]^2\}/R$$
$$+2(m_s\text{-}1)D(p_r^*)T_r^*(T_s\text{-}t_s)+(m_s\text{-}1)(m_s\text{-}2)$$
$$\times D(p_r^*)T_r^{*2}\}. \qquad (13)$$

The joint total profit for a special production cycle when the buyer decides to adopt a special order policy (denoted by $JTPS_2(m_s, p_s, t_s, T_s)$) can be obtained as the sum of the buyer's and the vendor's total profits.

$$JTPS_2(m_s, p_s, t_s, T_s) = TPBS_2(p_s, t_s, T_s)+TPVS_2(m_s, p_s, t_s, T_s). \qquad (14)$$

Similar as Case 1, if the vendor and buyer adopt its regular policies, then the joint total profit for a special production cycle will be divided into two periods (see Fig. 2). For the buyer, he/she orders Q^* units with the unit purchasing price v and sells with the retail price p^* in first replenishment cycle. At the following period, the buyer orders Q_r^* units at the unit purchasing price v_r and sells with the retail price p_r^*. Therefore, the total profit for the buyer in a special production cycle is given by

$$TPBN_2(m_s, T_s) = T^*TPB(p^*, T^*)$$
$$+[T_s+(m_s\text{-}1)T_r^*\text{-}T^*]\ TPB(p_r^*, T_r^*). \qquad (15)$$

As to the vendor, in first regular production cycle, the vendor products $Q^*+(m_r^*\text{-}1)Q_r^*$ units and then $m_r^*Q_r^*$ for the following production cycle. The total profit for the vendor in a special production cycle is

$$TPVN_2(m_s, t_s, T_s) = (v\text{-}c_r)D(p^*)T^*+(v_r\text{-}c_r)(m_r^*\text{-}1)D(p_r^*)T_r^*\text{-}S$$
$$\text{-}(h_vc_r/2)\{\{[D(p^*)T^*]^2\text{-}[(m_r^*\text{-}1)D(p_r^*)T_r^*]^2\}/R$$
$$+2(m_r^*\text{-}1)D(p_r^*)T_r^*T^*+(m_s\text{-}1)(m_s\text{-}2)D(p_r^*)T_r^{*2}\}$$
$$+[T_s\text{-}t_s+(m_s\text{-}m_r^*)T_r^*\text{-}T^*]/[D(p_r^*)/R+(m_r^*\text{-}1)T_r^*]$$
$$\times m_r^*T_r^*TPV_r(m_r^*, p_r^*, T_r^*). \qquad (16)$$

Consequently, the joint total profit for a special production cycle when the buyer decides to adopt a regular order policy (denoted by $JTPN_2(m_s, t_s, T_s)$) can be obtained as the sum of the buyer's and the vendor's total profits, which leads to

$$JTPN_2(m_s, t_s, T_s) = TPBN_2(m_s, T_s)+TPVN_2(m_s, t_s, T_s). \qquad (17)$$

Comparing (14) with (17), the joint total profit increase for Case 2 can be given by

$$g_2(m_s, p_s, t_s, T_s) = JTPS_2(m_s, p_s, t_s, T_s)\text{-}JTPN_2(m_s, t_s, T_s). \qquad (18)$$

where $JTPS_2(m_s, p_s, t_s, T_s)$ and $JTPN_2(m_s, t_s, T_s)$ are shown as in (14) and (17), respectively.

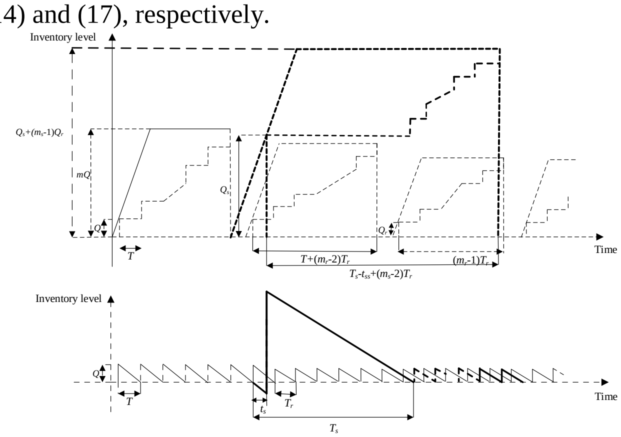

Fig. 2. The integrated inventory system when $Q_s/R>T^*$

IV. Theoretical Result

The objective of this problem is to determine the optimal pricing, ordering and production policies that correspond to maximizing the joint total profit increase. In order to solve this problem, we consider the following two cases: (i) $Q_s/R \leqq T^*$ and (ii) $Q_s/R>T^*$.

Case 1. $Q_s/R \leqq T^*$

Due to the high-power expression of the exponential function, we could not obtain a closed-form solution. Instead, for fixed p_s and T_s, we first check the effect of m_s on the joint total profit increase $g_1(m_s, p_s, T_s)$ in (11). Taking second-order derivative of $g_1(m_s, p_s, T_s)$ with respect to m_s, it gets

$$d^2\ g_1(m_s, p_s, T_s)/dm_s^{\ 2} = h_vc_rD(p_r^*)T_r^{*2}[D(p_r^*)/R\text{-}1]<0.$$

Hence, $g_1(m_s, p_s, T_s)$ is a concave function of m_s. Consequently, the search for the optimal number of shipments m_s (denoted by m_{s1}^*) is reduced to find a local maximum.

Next, for a given integer m_{s1}^*, we can prove that for any given retail price p_s, the optimal value of T_s not only exists but also is unique. And then for any given value of T_s, there exists a unique sell pricing p_s to maximize the objective function. The processes of proofs are similar to Dye [19], Wu et al. [10], Yang et al. [31], and hence are omitted here. Once the optimal retail price, p_s (denoted by p_{s1}^*), the length of special order cycle time, T_s (denoted by T_{s1}^*), and the number of shipment from the vendor to the buyer per production cycle, m_{s1}^*, are calculated, the optimal special order quantity, Q_s (denoted by Q_{s1}^*), and the joint total profit increase, $g_1(m_{s1}^*, p_{s1}^*, T_{s1}^*)$, can be obtained.

Case 2. $Q_s/R>T^*$

Similar as Case 1, for fixed p_s, t_s and T_s, we first check the effect of m_s on the joint total profit increase $g_2(m_s, p_s, t_s, T_s)$ in Equation (18). Taking second-order derivative of $g_2(m_s, p_s, t_s, T_s)$ with respect to m_s, it gets

$$d^2 g_2(m_s, p_s, t_s, T_s)/dm_s^{\,2} = h_v c_r D(p_r^*) T_r^{*2}[D(p_r^*)/R-1]<0$$

Hence, $g_2(m_s, p_s, t_s, T_s)$ is a concave function of m_s. Consequently, the search for the optimal number of shipments m_s (denoted by m_{s2}^*) is reduced to find a local maximum.

Next, for a given integer m_{s2}^*, we can prove that for any given retail price p_s, the optimal value of (t_s, T_s) not only exists but also is unique. And then for any given value of (t_s, T_s), there exists a unique sell pricing p_s to maximize the objective function. Once the optimal retail price, p_s (denoted by p_{s2}^*), the length of time, (t_s, T_s) (denoted by (t_{s2}^*, T_{s2}^*)), and the number of shipment from the vendor to the buyer per production cycle, m_{s2}^*, are calculated, the optimal special order quantity, Q_s (denoted by Q_{s2}^*), and the joint total profit increase, $g_2(m_{s2}^*, p_{s2}^*, t_{s2}^*, T_{s2}^*)$, can be obtained.

Finally, we develop a simple algorithm to illustrate the step-by-step solution procedure for finding the optimal solution as follows.

Algorithm

Step 1. Determine m^*, T^*, p^*, m_r^*, p_r^* and T_r^*, respectively.

Step 2. Find the optimal value $(m_{s1}^*, p_{s1}^*, T_{s1}^*, Q_{s1}^*)$ for Case 1.

Step 3. Find the optimal value $(m_{s2}^*, p_{s2}^*, t_{s2}^*, T_{s2}^*, Q_{s2}^*)$ for Case 2.

Step 4. Compare Q_{s1}^* with RT^*. If $Q_{s1}^*/R \leqq T^*$, then substitute the optimal value $(m_{s1}^*, p_{s1}^*, T_{s1}^*)$ into (11) to evaluate g_1^*; otherwise, set $g_1^*=-\infty$.

Step 5. Compare Q_{s2}^* with RT^*. If $Q_{s2}^*/R>T^*$, then substitute the optimal value $(m_{s2}^*, p_{s2}^*, t_{s2}^*, T_{s2}^*)$ into (18) to evaluate g_2^*; otherwise, set $g_2^*=-\infty$.

Step 6. Find $\underset{i=1,2}{Max}\, g_i$. Let $g^* = \underset{i=1,2}{Max}\, g_i$, then the optimal solution follows.

V. NUMERICAL EXAMPLE

To illustrate the optimal ordering policy, the following example is presented:

Given an inventory system with the following parameters: $D(p)=1000-8p$, where $p<125$, $R=1500$, $v=30$, $c=10$, $v_r=35$, $c_r=12$, $A=50$, $S=300$, $h_b=0.15$, $h_v=0.1$, $\pi=3$ in appropriate units. It is shown that the increase rates of unit production and supply price are $[(c_r-c)/c]\times100\%=20\%$ and $[(v_r-v)/v]\times100\%=16.67\%$, respectively. From the algorithm, we can obtain the optimal number of shipment, retail price, length of replenishment cycle time and order quantity as shown in Table 1.

TABLE I
THE OPTIMAL SOLUTIONS BEFORE AND AFTER PRICE INCREASE IN EXAMPLE 1

	Before supply price increase	After supply price increase	After price increase (special order policy)
Optimal number of shipment	6	6	15
Optimal retail price	67.9061	68.9466	69.0079
Optimal length of replenishment cycle time	0.2299	0.2132	0.3894
Optimal buyer's order quantity	105.024	95.591	174.446
Optimal vendor' s product quantity	630.144	573.546	2616.69

From Table 1, the buyer will reflect supply price increases on retail price with the rate $[(p_r^*-p^*)/p^*]\times100\%=1.51\%$ which is less than the increase rates of unit production and supply price. Furthermore, when the vendor announces a price increase that is effective starting on a particular future date, the optimal value of $g^*=g_1^*=82056.5-81705.6=350.882$ and the change rate on retail price is $[(p_s^*-p^*)/p^*]\times100\%=1.62\%$. From the economical viewpoint, the buyer will place a special order to take advantage of current lower purchasing cost before supply price increases. And then the vendor will also product more quantity and increase the number of shipment in a special production cycle.

VI. CONCLUSIONS

In this paper, we investigate a single-vendor, single-buyer integrated inventory model in response to announced supply price increase when market demand rate depends on the retail price. From the point of view of the vendor, he/she will determine the optimal shipment policy with the increase in the price of raw material. And then he/she will reflect the increasing cost on supply price (the buyer's purchase cost) and allow the buyer to make a special order. From the buyer's viewpoint, he/she will adopt a special order policy to determine the optimal special order quantity and retail price when demand rate depends on retail price. A simple algorithm to find the optimal solution is provided and a numerical example is presented to demonstrate the developed model and solution procedure. From the numerical results, we have that the buyer will place a special order to take advantage of current lower purchasing cost before supply price increases. And then the vendor will also increase product quantity and the number of shipment in a special production cycle.

ACKNOWLEDGMENT

The authors greatly appreciate the anonymous referees for their valuable and helpful suggestions regarding this paper. This research was partially supported by the National Science Council of the Republic of China under Grant NSC 102-2221-E-231-014.

REFERENCES

[1] E. Naddor, *Inventory Systems*. New York: Wiley, pp. 96–102, 1966.

[2] B. Lev, and A. L. Soyster, "An inventory model with finite horizon and price changes," *J. Oper. Res. Soc.*, vol. 30, pp. 43–53, January, 1979.

[3] S. K. Goyal, "A note on the paper: An inventory model with finite horizon and price changes," *J. Oper. Res. Soc.*, vol. 30, pp. 839–842, September, 1979.

[4] S. G. Taylor, and C. E. Bradley, "Optimal ordering strategies for announced price increases," *Oper. Res.*, vol. 33, pp. 312–325, March–April, 1985.

[5] S. K. Goyal, G. Srinivasan, and F. J. Arcelus, "One time only incentives and inventory policies," *Eur. J. Oper. Res.*, vol. 54, pp. 1–6, September, 1991.

[6] R. J. Tersine, "Economic replenishment strategies for announced price increases," *Eur. J. Oper. Res.*, vol. 92, pp. 266–280, July, 1996.

[7] A. K. Ghosh, "On some inventory models involving shortages under an announced price increase," *Int. J. Syst. Sci.*, 34, pp. 129–137, November, 2003.

[8] W. Huang, and V. G. Kulkarni, "Optimal EOQ for announced price increases in infinite horizon," *Oper. Res.*, vol. 51, pp. 336–339, April, 2003.

[9] S. Sharma, "On price increases and temporary price reductions with partial backordering," *Eur. J. Ind. Eng.*, vol. 3, pp. 70–89, March, 2009.

[10] K. S. Wu, L. Y. Ouyang, and C. T. Yang, "Coordinating replenishment and pricing policies for non-instantaneous deteriorating items with price-sensitive demand," *Int. J. Syst. Sci.*, vol. 40, pp. 1273–1281, December, 2009.

[11] M. A. Cohen, "Joint pricing and ordering policy for exponentially decaying inventory with known demand," *Naval Res. Logist. Quart.*, vol. 24, pp. 257–268, March, 1977.

[12] H. M. Wee, "Joint Pricing and Replenishment Policy for Deteriorating Inventory with Declining Market," *Int. J. Prod. Econ.*, vol. 40, pp. 163–171, August, 1995.

[13] P. L. Abad, "Optimal pricing and lot sizing under conditions of perishability and partial backordering," *Manage. Sci.*, vol. 42, pp. 1093–1104, August, 1996.

[14] H. M. Wee, and S. T. Law, "Replenishment and pricing policy for deteriorating items taking into account the time value of money," *Int. J. Prod. Econ.*, vol. 71, pp. 213–220, May, 2001.

[15] P. L. Abad, "Optimal pricing and lot-sizing under conditions of perishability, finite production and partial backordering and lost sales," *Eur. J. Oper. Res.*, vol. 144, pp. 677–686, February, 2003.

[16] S. Mukhopadhyay, R. N. Mukherjee, and K. S. Chaudhuri, "Joint pricing and ordering policy for a deteriorating inventory," *Comput. Ind. Eng.*, vol. 47, pp. 339–349, December, 2004.

[17] S. Mukhopadhyay, R. N. Mukherjee, and K. S. Chaudhuri, "An EOQ model with two-parameter weibull distribution deterioration and price-dependent demand," *Int. J. Math. Educ. Sci. Tech.*, vol. 36, pp. 25–33, August, 2005.

[18] H.H. Chang, J.T. Teng, L.Y. Ouyang and C.Y Dye, "Retailer's Optimal Pricing and Lot-sizing Policies for Deteriorating Items with Partial Backlogging," *Eur. J. Oper. Res.*, Vol. 168, pp. 51–64, January, 2006.

[19] C. Y. Dye, "Joint pricing and ordering policy for deteriorating inventory with partial backlogging," *Omega-Int. J. Manage. Sci.*, vol. 35, pp. 184–189, April, 2007.

[20] C. H. Su, "Optimal replenishment policy for an integrated inventory system with defective items and allowable shortage under trade credit," *Int. J. Prod. Econ.*, vol. 139, pp. 247–256, September, 2012.

[21] S.K. Goyal, "An integrated inventory model for a single supplier-single customer problem," *Int. J. Prod. Res.*, vol. 15, pp. 107–111, March, 1976.

[22] A. Banerjee, "A joint economic-lot-size model for purchaser and vendor," *Decis. Sci.*, 17, pp. 292–311, July, 1986.

[23] S. K. Goyal, "A joint economic-lot-size model for purchaser and vendor: a comment," *Decis. Sci.*, vol. 19, pp. 236–241, March, 1988.

[24] L. Lu, "A one-vendor multi-buyer integrated inventory model," *Eur. J. Oper. Res.*, vol. 81, pp. 312–323, March, 1995.

[25] S. K. Goyal, "A one-vendor multi-buyer integrated inventory model: A comment," *Eur. J. Oper. Res.*, vol. 82, pp. 209–210, April, 1995.

[26] R. M. Hill, "The optimal production and shipment policy for the single-vendor single-buyer integrated production-inventory problem," *Int. J. Prod. Res.*, vol. 37, pp. 2463–2475, November, 1999.

[27] S. K. Goyal, and F. Nebebe, "Determination of economic production-shipment policy for a single-vendor-single-buyer system," *Eur. J. Oper. Res.*, vol. 121, pp. 175–178, February, 2000.

[28] L .R. Khan, and R. A. Sarker, "An optimal batch size for a JIT manufacturing system," *Comput. Ind. Eng.*, vol. 42, pp. 127–136, April, 2002.

[29] S. L. Kim, and D. Ha, "A JIT lot-splitting model for supply chain management: Enhancing buyer-supplier linkage," *Int. J. Prod. Econ.*, vol. 86, pp. 1–10, October, 2003.

[30] H. Siajadi, R. N. Ibrahim, and P. B. Lochert, "Joint economic lot size in distribution system with multiple shipment policy," *Int. J. Prod. Econ.*, vol. 102, pp. 302–316, August, 2005.

[31] C. H. Yang, L.Y. Ouyang, and H. H. Wu, "Retailer's optimal pricing and ordering policies for non-instantaneous deteriorating items with price-dependent demand and partial backlogging," *Math. Probl. Eng.*, vol. 2009, pp. 1–18, December, 2009.

Chih-Te Yang is an Associate Professor in the Department of Industrial Management at Chien Hsin University of Science and Technology in Taiwan. He earned his Ph.D. from the Graduate Institute of Management Sciences at Tamkang University in Taiwan. His research interests are in the field of production/inventory control, and supply chain management. He has published articles in *International Journal of Production Economics, European Journal of Operational Research, Computers & Industrial Engineering, International Journal of Information and Management Sciences, Asia-Pacific Journal of Operational Research, International Journal of Systems Science, Mathematical Problems in Engineering, TOP, Journal of Scientific & Industrial Research, Central European Journal of Operations Research* and *European Journal of Industrial Engineering.*

Liang-Yuh Ouyang is a Professor in the Department of Management Sciences at Tamkang University in Taiwan. He earned his M.S. in Mathematics and Ph.D. in Management Sciences from Tamkang University. His research interests are in the field of Production/Inventory Control, Probability and Statistics. He has published in *Journal of the Operational Research Society, Computers & Operations Research, European Journal of Operational Research, Computers and Industrial Engineering, International Journal of Production Economics, IEEE Transactions on Reliability, Production Planning & Control, Mathematical and Computer Modelling, Applied Mathematical Modelling, Applied Mathematical and Computation, Journal of Global Optimization and International Journal of Systems Science etc*.

Yen-Wen Wang is an Associate Professor in the Department of Marketing and Distribution Management at Chien Hsin University of Science and Technology in Taiwan. He earned his Ph.D. in Industrial Engineering and Management at Yuan-Ze University in Taiwan. His research interests are in the field of production control and Management, Heuristics Optimization, Artificial Intelligent and Applications. He has published articles in *International Journal of Production Economics*, *Journal of Intelligent Manufacturing, International Journal of Production Research, Expert Systems with Applications, Decision Support Systems*.

Chun-Ming Yang is a Ph.D. Candidate in the Department of Management Sciences at Tamkang University in Taiwan. He earned his M.S. in Department of Business Administration from Asia University. His research interests are in the field of Process Capability Index and Decision Analysis. He has published in *International Journal of Production Research, Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, International Journal of Information and Management Sciences, and Advances in Information Sciences and Service Sciences*.