Quadratic Optimal Control of Coupling Roll and Yaw Spinning Palapa–B2R Satellite

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***Abstract*—**A successful implementation of Linear Quadratic Gaussian optimizer plus Full State Feedback Control (LQG-FSF) is achieved for coupling roll and yaw spin of Palapa B2R satellite model. The controller has been examined for response swiftness and step and square reference tracking. The aim is to achieve a robust spin stabilization of the satellite maneuver of coupling two-axis spins. Reasonable deadbeat coupling specifications are achieved despite slight overshoots of an order less than 4%. No more than 100 iterations have been imposed to achieve rise time of 12 sec, settling time of 30 sec and zero steady state error. Comparable coupling roll and yaw rates of 0.0175 rad/sec are found with control effort of yaw spinning thruster 1.196 times roll spinning due to inertia inequality effects.

***Keywords*—**Coupling roll and yaw spin, full state feedback control**,** linear Quadratic Gaussian optimizer, Palapa B2R satellite.

I. Introduction

SATELLITES are nowadays owned by many countries. The colony of space is typically for the applications of sciences, communication, weather forecast, Earth observation, navigation, astronomy, space physics and military services. By the end of past century, several attempts were seen to invade the space by Europe, Japan, China and India.

Spinning attitude control is one of the strategies for long life mission of more than 40 years. Proportional-Integral-Derivative (PID) controller is widely used in most of the satellite attitude controls [1]. The classical design usually fails for high-order systems and for multi-input multi-output (MIMO) systems [2]. The state space (SS) method is a simple technique to describe satellite spinning. In early 1960s Kalman introduced the ideas of state-variables, controllability and observability. Zadeh and Desoer had a significant impact in promoting the state-space method. Linear Quadratic Gaussian (LQG) optimal control was refined for aerospace areas during the 1960s and the 1970s [3]. The linear quadratic regulator (LQR) of an optimal control methodology has been applied in a wide range of applications [4]-[6]. The LQG minimizes quadratic cost function that trades off regulation performance and control effort.

This work implements a linear quadratic Gaussian (LQG) to optimize the full state feedback control (FSF) gains. The aim was set to attain satisfactory responses to inherently unstable Palapa B2R satellite spun around coupling axes. Finite-time optimal feedback controllers were applied for the nonlinear system representing a spacecraft detumbling problem with nonlinear constraints. The accuracy of the method was ascertained by the comparing the results with the open loop solutions [7]. Azor R. used optimal thrusters firing and horizon sensors for coupling roll-yaw control. It was found sufficiently adequate sensing and controlling only one of them directly and another one indirectly [8]. A MATLAB® framework of M-file batch and SIMULINK® is used to model, simulate and control coupling spinning satellite. Designing for one loop at a time is inadequate for this level of cross-coupling MIMO design to correctly handle coupling effects. However, the direct MIMO design for multivariable satellite is implemented. Some degree of cross-coupling spinning satellite is obtained for yaw and roll stability. Physically, two-axes spinning mode has to be performed in the form of joint-axis stability. The response reveals a severe deterioration in regulation performance.

II. Method

A. Satellite Model

The Palapa B2R satellite was launched in 1984 with mass of 1200 kg by Indonesia's domestic satellite system. It is 6.83 m high and 2.16 m diameter. It provides regional communications to the country's inhabited islands and increased coverage including the Philippines, Thailand, Malaysia and Singapore. Briefly, it is described as cylindrical structure, spin stabilized, hydrazine propulsion system for attitude control, body mounted solar cells of 1060w and despun antenna platform. The reason of choosing this model mainly refers to the availability of satellite data. Moreover, the analysis made in this work could be correlated to the satellite still in service.

The satellite is assumed inflexible and will not experience space disturbances. On the basis of the principle axes theory the satellite mass moment of inertia matrix is given by [9].

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$$\vec{I} = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} 1250 \\ 1500 \\ 1800 \end{bmatrix} \; kgm^2$$

where I_x, I_y and I_z are mass moments of intertie around X, Y and Z axes. I_{xy}, I_{yz} and I_{zx} are mass product moment of inertia. The satellite is inherently unstable and cannot keep balance to initial orientation without controller.

The Palapa B2R uses rotor spinning in order to stabilize platform and antenna. Control moments are produced by the angular acceleration and deceleration of the rotor's spin. Because of the configuration of satellites' inertia, the satellite's motion in the yaw mode is coupled with its roll mode. With those couples, the satellite's attitude can be controlled by the angular acceleration and deceleration of the rotor that spun in pitch mode. The controlled satellite plant can be considered a linear multivariable system. In practice, the optimal control based on quadratic performance index is quite popular. For optimal control system design, the main task is to choose control vector (u) which should make the performance index to be minimum. As known if the integral limit of quadratic performance index is from 0 to infinite, the control law will be linear, i.e.,

$$u(t) = -\mathrm{K}\,x(t) \tag{1}$$

Therefore, the design of quadratic performance index based optimal control system is equal to find out all elements of steady feedback matrix $\mathbb{K}$.

A. Dynamic Model

Because the development of a physical model for a satellite is lengthy, the SS equations can be derived from the coupling-spinning state satellite attitude kinematics equations below:

$$\left.\begin{aligned} T_x(t) &= I_x\alpha_x - (I_y - I_z)\omega_y\omega_z \\ T_y(t) &= I_y\alpha_y - (I_z - I_x)\omega_z\omega_x \\ T_z(t) &= I_z\alpha_z - (I_x - I_y)\omega_x\omega_y \end{aligned}\right\} \tag{2}$$

where T_x, T_y and T_z are torques around X, Y and Z axes respectively. α_x, α_y *and* α_z are angular accelerations around X, Y and Z axes respectively. ω_x, ω_y *and* ω_z are angular velocities around X, Y and Z axes respectively.

The angular kinematics and the coupling-spinning state satellite attitude kinematics equations are as below:

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot{\varphi} - \omega_0\psi \\ \dot{\psi} + \omega_0\varphi \\ \dot{\theta} - \omega_0 \end{bmatrix} \tag{3}$$

where φ, ψ and θ are roll, yaw and pitch Euler angle due to orbit coordinate system spinning around X, Y and Z axis respectively. Nutation damping (ω_0 = 0.0012 rad/sec at Earth equator) is due to the nutational motion of the satellite moments of inertia and momentum wheel.

Since orbit angular velocity is considerably small and the pitch spin is ignored and (3) turns out as

$$\begin{aligned} T_x(t) &= I_x\ddot{\varphi} - (I_y - I_z)\dot{\theta}\dot{\psi} + (I_y - I_z)\omega_0\dot{\psi} \\ T_y(t) &= I_y\ddot{\psi} - (I_z - I_x)\dot{\varphi}\dot{\theta} + (I_z - I_x)\omega_0\dot{\varphi} \\ T_z(t) &= I_z\ddot{\theta} - (I_x - I_y)\dot{\psi}\dot{\varphi} \end{aligned} \tag{4}$$

Also, the terms multiplication with roll and/or yaw Euler angles may be neglected. Thus Eq. (4) is rewritten as

$$\left\{\begin{aligned} T_x(t) &= I_x\ddot{\varphi} + (I_z - I_y - I_x)\omega_0\dot{\psi} + (I_y - I_z)\omega_0{}^2\varphi \\ T_y(t) &= I_y\ddot{\psi} + (I_y + I_x - I_z)\omega_0\dot{\varphi} + (I_z - I_x)\omega_0{}^2\psi \\ T_z(t) &= I_z\ddot{\theta} \end{aligned}\right\} \tag{5}$$

The state equation of linear time-invariant system was assumed. The rigid satellite attitude dynamic equation of yaw and roll is:

$$\begin{aligned} T_x(t) &= I_x\ddot{\varphi} + (I_z - I_y - I_x)\omega_0\dot{\psi} \\ T_y(t) &= I_y\ddot{\psi} + (I_y + I_x - I_y)\omega_0\dot{\varphi} \end{aligned} \tag{6}$$

Highly nonlinear characteristics of satellite dynamic system are linearized in a form of double-integral plant.

B. SS Model

The linear time-invariant SS system can be given by state-variable and output equations respectively

$$\left.\begin{aligned} \dot{x}(t) &= \mathrm{A}x(t) + \mathrm{B}_u u(t) + \mathrm{B}_\omega\,\omega(t) \\ y(t) &= \mathrm{C}_y x(t) \end{aligned}\right\} \tag{7}$$

where A is satellite matrix, B_u is control input matrix, B_ω is disturbance matrix, ω is satellite disturbance and u is control effort. C_y is output matrix.

The measurements available for feedback are

$$m(t) = \mathrm{C}_m x(t) + v(t) \tag{8}$$

where C_m is measurement matrix and v is measurement noise.

Output noise and external disturbances such as solar pressure radiation will be examined in future work. Thus, the third term of (7) and the second term of (8) may be conveniently missed out. Once the system is linearized the step response can be derived for any input amplitude. The state-variable method is used to describe the satellite dynamics. A set of first-order differential equations in the vector-valued state of the system was composed.

The second term of (6) may be reduced by below assumptions

$$\varrho = (I_z - I_y - I_x)\omega_0 \tag{9}$$

Zero state initial conditions can be applied to promptly discard transition solution. The state space expressions can be written as below

$$\begin{cases} x = [\dot{\varphi} \quad \varphi \quad \dot{\psi} \quad \psi]^T \\ y = [\dot{\varphi} \quad \varphi \quad \dot{\psi} \quad \psi]^T \\ u = [T_x \quad T_y]^T \\ x_0 = [0 \quad 0 \quad 0 \quad 0]^T \text{ (initial state)} \end{cases}$$

The associated SS matrices which were used to simulate Palapa B2R satellite are shown below.

$$\mathrm{A} = \begin{bmatrix} 0 & 0 & -\varrho/\mathrm{I}_x & 0 \\ 1 & 0 & 0 & 0 \\ \varrho/\mathrm{I}_y & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathrm{B}_u = \begin{bmatrix} 1/\mathrm{I}_x & 0 \\ 0 & 0 \\ 0 & 1/\mathrm{I}_y \\ 0 & 0 \end{bmatrix}$$

$$\mathrm{C}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathrm{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathrm{C}_m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C. Linear Quadratic Gaussian Algorithm

The LQG control methodology, known as H_2 control, provides a means of systematic designing of optimal controller for high-order systems via optimisation. Kalman filter may be used to estimate all the state variables $\hat{x}(t)$ to form linear feedback controller

$$u(t) = -\mathrm{K}\hat{x}(t) = -\begin{bmatrix} \mathrm{K}_{11} & \mathrm{K}_{12} & \mathrm{K}_{13} & \mathrm{K}_{14} \\ \mathrm{K}_{21} & \mathrm{K}_{22} & \mathrm{K}_{23} & \mathrm{K}_{24} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} \quad (10)$$

The typical version of the Kalman filter equation can be shown as

$$\dot{\hat{x}}(t) = \mathrm{A}\hat{x}(t) + \mathrm{B}_u u(t) + \mathrm{G}(t)(m(t) - \mathrm{C}_m \hat{x}(t)) \quad (11)$$

The initial state was taken as

$\hat{x}(0) = [\text{zeros}]_{4\times1}$

The Kalman gain G(t) is obtained by solving a pair of equations.

$$\left.\begin{aligned} \dot{\Sigma}_e(t) &= \Sigma_e(t)\mathrm{A}^T + \mathrm{A}\Sigma_e(t) + \mathrm{B}_\omega \mathrm{S}_\omega \mathrm{B}_\omega^T \\ &-\Sigma_e(t)\mathrm{C}_m^T \mathrm{S}_v^{-1}\mathrm{C}_m\Sigma_e(t) \\ \mathrm{G}(t) &= \Sigma_e(t)\mathrm{C}_m^T \mathrm{S}_v^{-1} \end{aligned}\right\} \quad (12)$$

Subjected to the initial condition

$\Sigma_e(0) = [\text{zeros}]_{4\times4}$

where $\Sigma_e(t)$ is estimation of error covariance matrix, S_v and S_ω are plant noise and disturbance spectral densities respectively. Superscripts -1 and T signify inversion and transpose operations respectively. The optimal performance index can be expressed by the quadratic cost function

$$J(x(t), u(t)) = E\left[\frac{1}{2}x^T(t_f)S(t)x(t_f) + \frac{1}{2}\int_0^{t_f}[x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)]dt\right] \quad (13)$$

where $S(t)$ and $Q(t)$ are positive semi-definite weighting matrices of state variables. $R(t)$ are positive definite weighting matrices of control vector. Trial and error may be used to find these matrices. t_f is final time. The Hamilton function is

$$H = -\frac{1}{2}[x^T(t)\mathrm{Q}(t)x(t) + u^T(t)\mathrm{R}(t)u(t)] + \lambda^T(t)[\mathrm{A}x(t) + \mathrm{B}_u u(t)] \quad (14)$$

Q(t) = Q and R(t) = R for steady considerations. The quadratic optimal controller design matrices were chosen to weight the yaw and roll equally and control as follows

$\mathrm{Q} = diag[0 \quad 800 \quad 0 \quad 1000], \ \mathrm{R} = I_{2\times2}$

The optimal control effort can be found as

$$u^*(t) = \mathrm{R}^{-1}\mathrm{B}_u^T\lambda(t) \quad (15)$$

where control parameter and the state feedback gain can be found by

$$\left.\begin{aligned} \lambda(t) &= -\mathrm{P}(t)\hat{x}(t) \\ \mathrm{K}(t) &= \mathrm{R}^{-1}\mathrm{B}_u^T\mathrm{P}(t) \end{aligned}\right\} \quad (16)$$

Matrix P(t) could be worked out by Riccati equation

$$\dot{\mathrm{P}}(t) = -\mathrm{P}(t)\mathrm{A} - \mathrm{A}^T\mathrm{P}(t) + \mathrm{P}(t)\mathrm{B}_u\mathrm{R}^{-1}\mathrm{B}_u^T\mathrm{P}(t) - \mathrm{Q} \quad (17)$$

Instead algebraic Riccati equation may be used for steady results

$$0 = -\mathrm{PA} - \mathrm{A}^T\mathrm{P} + \mathrm{PB}_u\mathrm{R}^{-1}\mathrm{B}_u^T\mathrm{P} - \mathrm{Q} \quad (18)$$

Subjected to the final condition

$$\mathrm{P}(t_f) = \mathrm{H} \quad (19)$$

A. Model and Simulation

The coupling spinning satellite equations can be rearranged as

$$\varphi(s) = \frac{T_x(s)}{I_x s^2} - \frac{(I_z - I_y - I_x)\omega_0 s\psi(s)}{I_x s^2} \quad (20)$$

$$\psi(s) = \frac{T_y(s)}{I_y s^2} - \frac{(I_x - I_z + I_y)\omega_0 s\varphi(s)}{I_y s^2} \quad (21)$$

The implementations of LQG controller for coupling spins are schematically shown in Fig. 1.

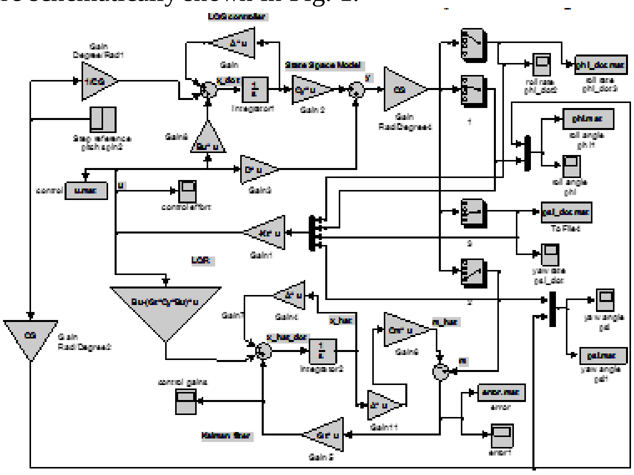

Fig. 1 Simulink model of LQG control of coupling roll and yaw Palapa B2R satellite

III. Results and discussions

A balanced state-space model of coupling spin satellite was considered using diagonal similarity transformation. Those dynamic parameters were found based on the conditions of controllability and observability. Two conditions were satisfied for a convergent Kalman estimator of the satellite model and covariance data.

1) All unstable poles of A were observable through C.
2) Q and R were selected so that

$$\begin{bmatrix} B_u & 0 \\ C_y & I \end{bmatrix} \begin{bmatrix} Q & N \\ N' & R \end{bmatrix} \begin{bmatrix} B_u & 0 \\ C_y & I \end{bmatrix}' \text{ is positive definite}$$

The optimisation problem aimed to minimise (13) subjected to (7) where the matrix N was set to zero in order to realize steady solution of the algebraic Riccati equation. Satellite spin rate was taken comparable to the orbital angular velocity for convenient Earth observation missions. Roll and yaw thrusts *u1* and *u2* were manipulated by controller to simultaneously track roll and yaw coupling set points. The model consisted of six transfer functions correlating each state variable to controller efforts. Next the responses of coupling roll and yaw spins of Palapa B2R satellite are assessed based on step and square commands.

A. Time Step Responses

Coupling roll and yaw attitudes of Palapa B2R satellite are shown in Fig. 2. To show transient response a close up view of coupling step roll and yaw attitudes is given in Fig. 3. Overall, satisfactory convergences of coupling roll and yaw attitudes were achieved for the Palapa B2R satellite. However, dissimilar synchronization of two attitudes was encountered particularly before 30 sec of simulation time. Several techniques were applied to obtain deadbeat coupling spin responses. Such design merits could not be realised particularly for negligible overshoot qualities. The lowpass filter in series with the closed-loop system was also endeavoured to eradicate a little overshoot of 4% of the coupling spin satellite. Both attitudes reached comparable steady state based on ±2% criterion at approximately 30 sec. However, roll spin was slightly swifter and higher overshot than yaw spin. Table II summaries the time response attributes of coupling roll and yaw spins of Palapa B2R satellite.

Deadbeat optimal responses were achieved using proportional regulator with gains of 3.3192 and 2.805 for roll and yaw attitude respectively. The LQGR dampened oscillations for inherently unstable coupling spinning satellite.

Coupling step roll and yaw rates of Palapa B2R satellite are shown in Fig. 4. Steady convergences were reached for coupling roll and yaw rates of 0.0175 rad/sec at 30 sec. The controller did well to achieve optimal coupling in two axes as shown in Fig. 5. Control effort of yaw spinning thruster was 1.196 times roll spinning due to inertia effects.

TABLE II
Time response attributes of coupling roll and yaw spin

Spin	Rise Time (sec)	Peak Time (sec)	Overshoot %
Roll	12.06	15.99	3.8
Yaw	12.27	16.21	1.97

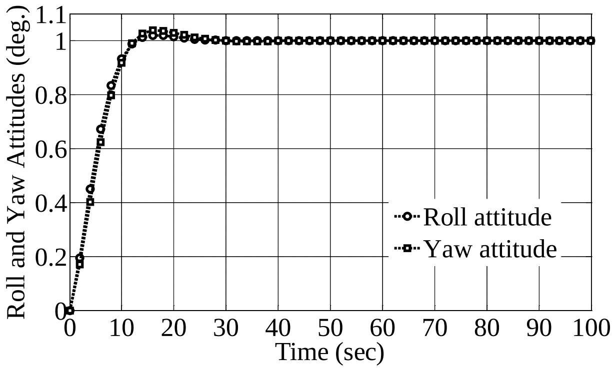

Fig. 2 Coupling step roll and yaw attitudes of Palapa B2R satellite

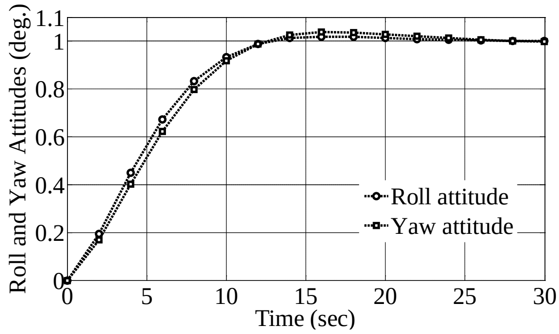

Fig. 3 A close up view of coupling step roll and yaw attitudes

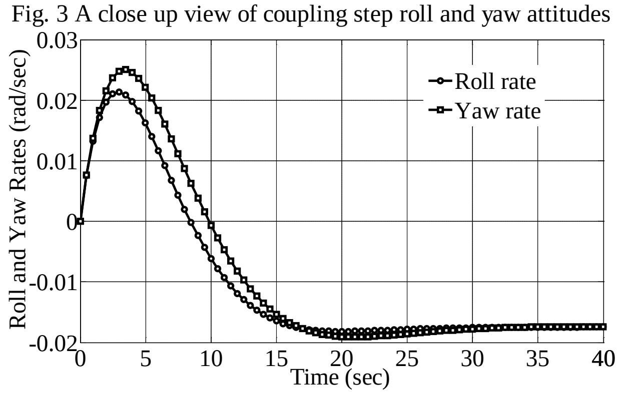

Fig. 4 Coupling step roll and yaw rates of Palapa B2R satellite

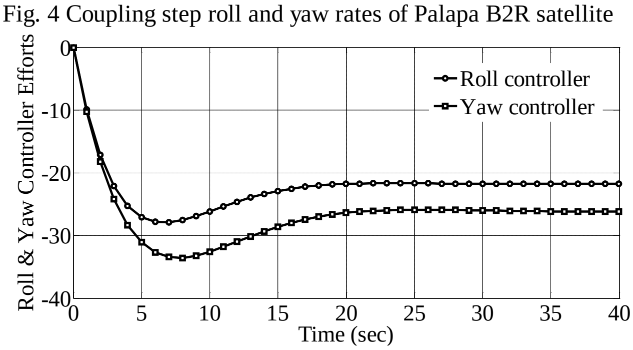

Fig. 5 Controller efforts of coupling step roll and yaw of Palapa B2R

B. *Square-Waved Responses*

A square signal reference of amplitude of one deg. and frequency of 0.01 Hz was also implemented for further analyses and verifications. Because of high nonlinearity and coupling effects associated with spinning roll and yaw satellite a saturation object that stores the saturation nonlinearity for estimating Hammerstein-Wiener models was implemented. Figs. 6 and 7 show a comparison without and with saturation of square roll and yaw attitude of Palapa B2R satellite respectively. Clearly, saturation nonlinearity hooker eliminated small overshoot seen before implementing saturation. Reasonable following square reference signal were obtained for roll and yaw coupling spins.

Fig. 8 shows coupling square roll and yaw rates of Palapa B2R satellite. The square tracking response was repeated at period of 100 sec. Similar to step response, steady convergences were reached for coupling roll and yaw rates of 0.0175 rad/sec. However, short divergent roll and yaw rates of 0.0675 rad/sec were seen at 54.17 sec and 102.32 sec or in comparable periods. Fig. 9 shows controller efforts of coupling square roll and yaw responses of Palapa B2R satellite. The controller efforts proved to achieve optimal coupling in two axes apart from peaks of thrust produced at 54.17 sec and 102.32 sec or equivalent periods.

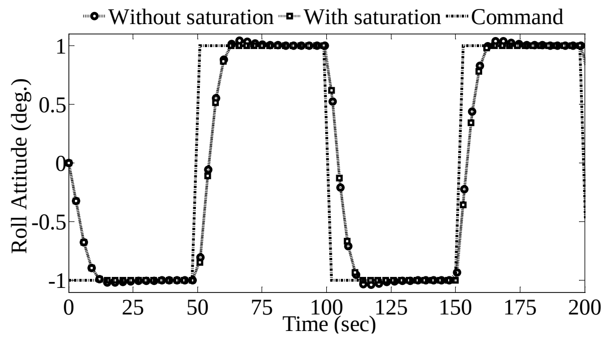

Fig. 6 Square roll attitude of Palapa B2R satellite without and with saturation

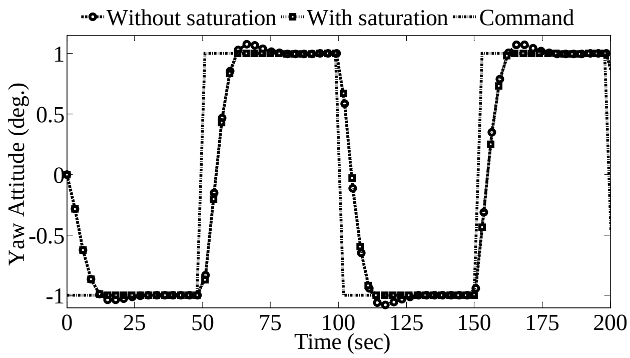

Fig. 7 Square yaw attitude of Palapa B2R satellite without and with saturation

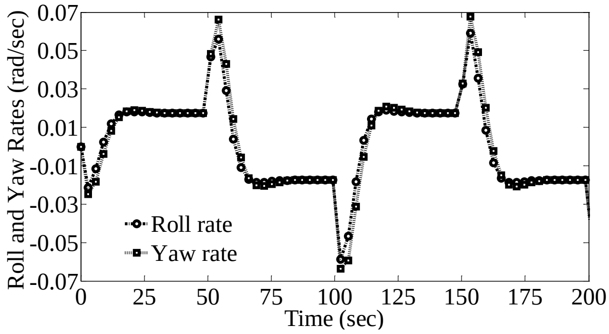

Fig. 8 Coupling square roll and yaw rates of Palapa B2R satellite

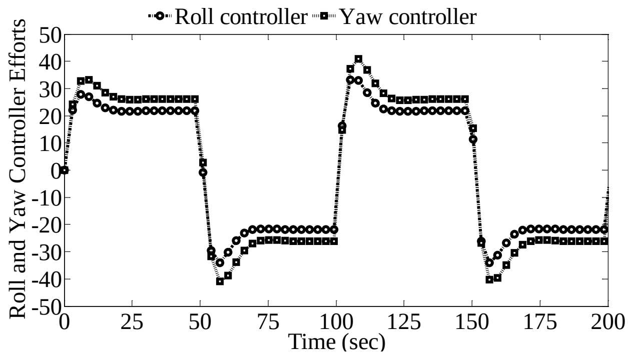

Fig. 9 Controller efforts of coupling square roll and yaw responses of Palapa B2R satellite

IV. Conclusion

Clearly, the LQG performed well with reasonable gains. The simulation and analytical results have agreed well. Seemingly, the static gains are more than adequate. However, the terminal conditions are important in short-time horizon cases that the time-varying gains should be used.

Robustness may be improved using loop transfer recovery procedure which should be appeared in the LQG cost function or adding classical integral to eliminate disturbance effects for good tracking performance. Also, H_∞ design may be seen to compare the results.

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