

# The Study of Stress State in Indentation of a Flat Punch with Rounded Edge in Axisymmetric Backward Extrusion

Ban Bakir Alamer<sup>1</sup>

**Abstract**—Analytical solution have to obtained of contact problem for indentation of flat punch with rounded edges in axisymmetric extrusion to both surface pressure and interior stress fields for a given die shape at different velocity field. Both sliding friction and partial slip condition must be concluded in the equation of metal flow for extruded material taking into consideration vonmises yield criterion and the complicated high elastic contact stress in the flat and rounded edges dies . An experimental method using prototype sample of photo elastic material for configuration the effect of rounded edges on the stress distribution in the contact zone . A numerical method used programming for solving the analytical equations of metal flow using quick basic and comparing the theoretical with the experimental results .A kinematic ally admissible metal flow cylindrical velocity using strain rate of plastic flow has been proposed to obtain the load of contact on the elastic punch face during the axi-symmetric backward extrusion. The theoretical results predict the forming force , rule of friction , area reduction and the metal flow quite satisfactorily when compared with the experimental results.

**Keywords**— Backward extrusion, Contact slip, Indentation, Photo elastic, Sliding friction.

## I. INTRODUCTION

**I**N cold forming of axi-symmetric backward extrusion , the cold forming more and more components are improved in quality by product a suitable loading for pressing the die against the plastic metal . in these problems contacts arise that characteristically have a flat base with some kind of radius at the edge which is either pre-existing or generated by wear. The complexity of this problem is the contact pressure between an elastic face with sliding friction and the surface of plastic metal , so that the design procedure for these contacts is usually one derived from experience as it is notoriously difficult to determine the contact pressure in contrast to the hertzian contact where simple closed form solutions exist between two elastic bodies .Often the sides of the punch are straight and normal to the free end and a deduction of the contact stress state induced is hot as height forward matter .[5]

Timoshenko and goodier [8] assumed that the punch itself is rigid and is pressed into a compliant half-plane . The contact deformation is therefore accommodated entirely

within the latter and a classical formulation may be employed .Khdem etal [1] analyzed the contact pressure using an elastic formulations that are appropriate to a rectangular domain where the geometry of the punch has a significant effect on the contact pressure distribution which observed experimentally .M.M. Moshksar etal [2] , [3] have presented an upper bound solution for the early and final stages of backward extrusion forging of hollow components . S.B.Biner [6] has proposed an analytical solution of axi-symmetric backward extrusion for the study of metal flow in this process using finite element method (FEM) and model material techniques . Also Sackfiled etal [7] have proposed a velocity field for the backward forward combined extrusion of polygonal cup – bar shape components using upper bound theorem and visio plasticity method for experimental verification.

In this study, the effect of the circular parts of the geometry and the radius of the corners of the punch on the contact stresses distribution between the elastic surface of the punch, the plastic material flow and different strain rates  $\dot{\epsilon}$  and the velocity of the axi-symmetric extrusion which must be analyzed by using metal flow theory with contact pressure laws and the experimental results for verification of the theoretical results.

## II. THEORETICAL ANALYSIS

The general solution in the case of a symmetrical complete or incomplete contact over the range  $-b \leq x \leq b$  as shown in fig(1) is given by Mushkelishvili's [4] inverse of the singular equation as :-

$$p(x) = - \int_{-b}^b \frac{p(t) dt}{x-t} \quad (1)$$

Here, both  $p(b)$  and  $p(-b)$  are bounded (incomplete contact) and it may be taken as  $p(b) = 0$  which reduces the general solution for contact to :

$$p(x) = \int_{-b}^b \frac{h'(t) dt}{x-t} \quad (2)$$

Where,  $h'(t)$  =the slope at each point on the profile of the punch

$$= (1 - \nu_1^2) + (1 - \nu_2^2) \quad (3)$$

<sup>1</sup> Ban Bakir Alamer, University of Baghdad –Collage of engineering – Mechanical Department (e-mail: ban\_msc\_eng@yahoo.com ).

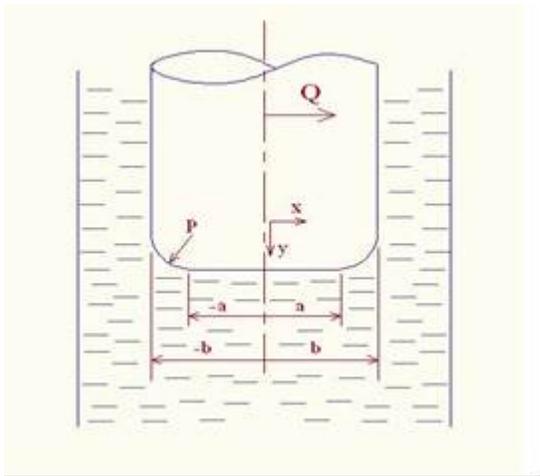


Fig. 1 Geometry of the punch in axi symmetric extrusion

The applied load  $p$  is in equilibrium with the pressure distribution and is given by :[1]

$$p = - \int_{-b}^b p(x) dx = - \frac{E^*}{2} \int_{-b}^b \frac{hB(t).t dt}{\sqrt{b^2+t^2}} \quad (4)$$

Here, we take  $h'(x) = -(a+x)/R$  if  $-1 \leq x \leq a$   
 $h'(x) = 0$  if  $-a \leq x \leq a$   
 $h'(x) = -(x-a)/R$  if  $a \leq x \leq b$

It may be found using overall equilibrium as a function of the load  $p$  as (8)

$$\frac{2PR}{a^2 E^*} = \frac{\pi - 2\phi^0}{4 \sin \phi^0} - \frac{\cot \phi^0}{2} \quad (5)$$

Where  $\phi^0 =$  the angle specifying the contact half width  $b$   $\phi$  = the angle of contact at any point on the contact length

Example :  $-\pi/2 \leq \phi \leq \pi/2$  is corresponds to  $-b \leq x \leq b$  In particular after the normal load  $p$  is applied ,  $p$  is kept constant and a monotonically increasing tangential force  $Q$  to cause sliding and the metal flow under the punch face with stick and slip zones depend on the displacement and velocity of particles parallel with surface to the surface traction so that the equation of sliding can be written as (7)

$$\frac{E^*}{2} g'(x) = \frac{1}{\pi} \int_1^{\lambda} \frac{q(\lambda) d(\lambda)}{x-\lambda} \beta(\lambda) \quad (6)$$

Where  $g(x)$  is the relative tangential displacement of surface particles  $g'(x) = dg(x) / dx$  its derivative Tangential equilibrium will be satisfied if :

$$Q = \int_1^{\lambda} q(\lambda) d\lambda \quad (7).$$

### III. METAL FLOW THEORY

The values of  $p(t)$  and  $q(\lambda)$  at any point along the contact region could be obtained according to st- venant which the plastic flow phenomenon can be condensed into two tensor equations [1]

$$\left. \begin{aligned} \sigma'_{ij} &= 2\lambda \epsilon'_{ij} \\ \epsilon'_{ij} &= 0 \end{aligned} \right\} \quad (8)$$

$$\lambda = \tau_{oct} / \gamma'_{oct} \quad (9)$$

So, eq (8) and (9) in the expanded form in the principal coordinate system can be written as :-

$$\begin{bmatrix} \frac{2\sigma_1 - \sigma_2 - \sigma_3}{a} & 0 & 0 \\ 0 & \frac{2\sigma_2 - \sigma_1 - \sigma_3}{a} & 0 \\ 0 & 0 & \frac{2\sigma_3 - \sigma_1 - \sigma_2}{a} \end{bmatrix} = \frac{2\lambda}{dt} \begin{bmatrix} dt_1 & 0 & 0 \\ 0 & dt_2 & 0 \\ 0 & 0 & dt_3 \end{bmatrix} \quad (10)$$

So that from the strain rate given and substituted in equation (10) we can obtain the values of the principle stresses and then the values of  $p$  and  $q$  as :-

$$\left. \begin{aligned} \sigma_1 &= \frac{p}{2} + \sqrt{\left(\frac{p}{2}\right)^2 + Q^2} \\ \sigma_2 &= \frac{p}{2} - \sqrt{\left(\frac{p}{2}\right)^2 + Q^2} \end{aligned} \right\} \quad (11)$$

From equation (4) and equation (7) of the values of  $p(x)$  and  $q(x)$  at every point along the contact length could be obtained.

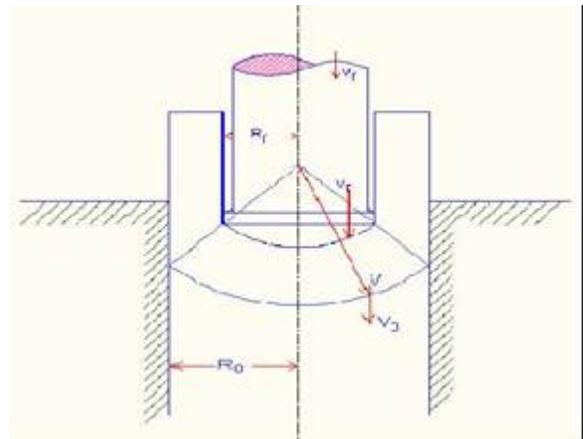


Fig. 2 Velocity field for the early stage of axi symmetric backward extrusion or piercing (transformed velocity field)

We use materials called perfect rigid plastic material which mean that the metal when reaches the plastic region the plastic deformation become under constant stress therefore during using such metals we can represent them under load equal to yield stress then it can study the stresses distribution on the punch.

TABLE I  
UNITS AND SYMBOLES

symbol	Quantity	units
b	Symmetrical complete contact for the indenter	mm
a	Symmetrical incomplete contact for the indenter	mm
x	The direction of slip regions	
E	Young modulus	KN/mm <sup>2</sup>
h	The slope of indenter at each point	
t	Domain of integration	
v	Velocity of indenter in the y direction	m/s
R	The radius of curvature of the corners	mm
p	The applied load	kN
λ	The domain of integration	
φ	The angle of contact (auxiliary angle )	degree
q	Applied local load on the domain λ	kN
Q	Shearing force traction	kN
σ	Normal stress	KN/mm <sup>2</sup>
β	Dundurs constant	
ε̇	deviator of strain rate tensor	
φ	the angle specifying the contact half width b	
w	The width of the specimen	mm
p	The applied force	kN
T	The thickness of the specimen	mm
K	The relative stress	KN/mm <sup>2</sup>
N	The fringe number	
σ <sub>1</sub> , σ <sub>2</sub>	The principles stresses	KN/mm <sup>2</sup>



Fig (3) The pattern of the extrusion punch with load= 0 Kg

#### IV. EXPERIMENTAL ANALYSIS:

The experimental method used is photo elasticity and the profile and external dimension of the joint is tall exactly 2<sup>1/2</sup> times the size of the external of facilitate handling and data collection.

This model was a two dimensional representation of a three – dimensional structure , with the profiles being taken in the axial radial plane of the three dimensional component .As the loading required for these models was purely axial no special loading frame was required [9] .

In photo elasticity the value of the stresses can be found by the equation:

$$\sigma_1 - \sigma_2 = \frac{CN}{t} \tag{12}$$

We could write equation (7) as:

$$\frac{p}{N} = CW$$

The fringe pattern can be shown Fig (3) for different loads.

The properties of the photo elastic material is given by the material are CR – 39 which has :

When using die with elastic metal with pattern with perfectly plastic metal it can easy known the type of the load applied that lead to obtain the yield stress of the die metal and can then comparing it with the yield stress for elastic material because the stress applied on patterns will still constant after yield stress. [4]

TABLE II  
CONSTANT VALUES

Tensile strength	Young modulus	Passions ratio	Value of the fringe order
210.9 kg/cm <sup>2</sup>	21 kg/cm <sup>2</sup>	0.42	C=7 kg/cm <sup>2</sup>



Fig. 4 The pattern of the extrusion punch with load= 3.5 Kg



Fig. 5 The pattern of the extrusion punch with load= 5.5 Kg



Fig .6 The pattern of the extrusion punch with load= 6.5 Kg

When reduce the diameter of punch 1 mm:

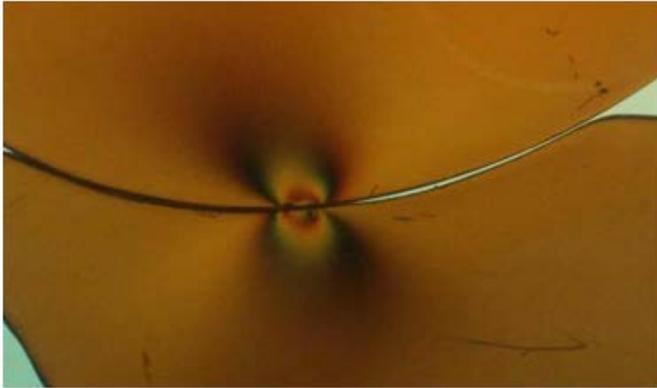


Fig. 7 The pattern of the extrusion punch with load= 3.5 Kg

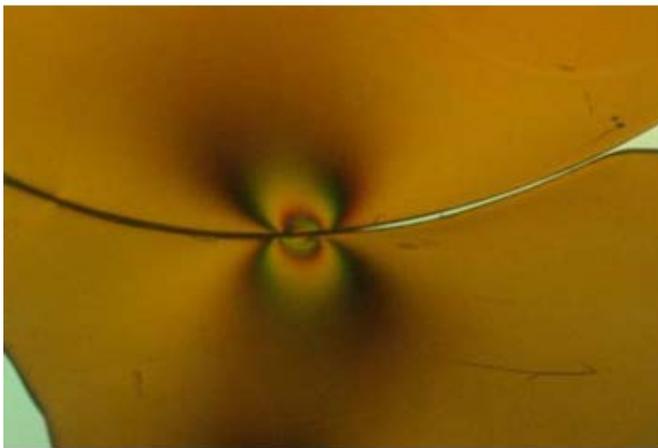


Fig. 8 The pattern of the extrusion punch with load= 5

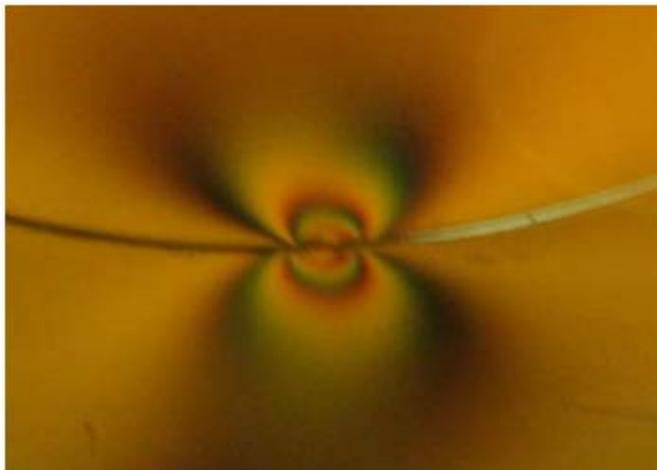


Fig .9 The pattern of the extrusion punch with load= 8.5 Kg

## V. RESULTS AND DISCUSION

Strain flow was obtained from equation ( 2 ), (3), where we consider the metal as soon as access to the pressure resulting from compress metal, the metal will deform plastically because it is rigid perfectly plastic .

Fig. (10) shows the effect of the width of the punch  $a/b$  and the contact pressure  $p$  for a constant 70% backward reduction in area. It can be shown that the max value of  $p$  was obtained near the rounded edge of the punch and was increased with increasing the flatness of the punch  $a$  with respect to total contact  $b$  and some decreasing in the center of the punch because of metal flow increasing in this point [6].

When punch exceed during extrusion process ,the first problem that facing the extrusion was transformation through the final end of the die where area of dead zone was generated and after that the metal flow become with low load from the first .

Fig. (11) illustrate the effect of increasing strain rate and relationship with the velocity of advances of the punch for different ratios of  $a/b = 0.1$  and  $0.5$  .It can be shown that by decreasing the flatness of the punch the velocity of the metal flow increases which causes an increase in the value of the slip forces  $Q$  causes more increased in the extrusion force.

Fig. (12) and Fig. (13) shown the increasing of flow stresses with the velocity of the punch for percentage reduction of 4% and 8% . This relation illustrate that with increasing the ratio of contact  $a/b$  the max flow stress will be increased so that the pressure required for the extrusion force is decreased as the punch advances but the smaller the backward area reduction , the more gradual is the decrease of extrusion force . this can be seen very clearly when increasing the reduction of area from 4% to 8% .

We can calculate velocity of punch ( $v$ ) for every value by using strain rate ( $\dot{\epsilon}$ ) and thickness of metal ( $h$ ) by using equation  $\dot{\epsilon}$  and we use to take mean velocity because of the dead zone therefore strain rate will be mean also.

When comparing the theoretical stress distribution along the contact length  $x/b$  with the experimental values that obtained from photo elastic analysis it can be seen from Fig. 14.(A) and Fig. 14 (B) the percentage error is between 8% to 10% for max reduction in area which could be found that the stress distributed on the punch contact have been affected by the strain rate of the material from 4% to 8% and on the contact ratio  $a/b$  which could be obtained by suitable rounded of the punch edge .

## VI. CONCLUSION

The method of analysis proposed in this paper offers a method of obtaining the stress on the punch and the solution for the effect of strain rate and contact ratio on the stress flow of the metal in axi symmetric backward extrusion.

Different velocity fields and strain rates have been used to obtain the effect of the rounding of the punch edges on the distribution of max principle stress . This relation is dependent on the s.venant metal flow and Mushkelishivs equation.The results shown that with increasing the contact ratio  $a/b$  the normal applied load will be increased and this will increased with increasing the percentage of reduction

area. The theoretical results give good agreement with the experimental using photo elastic material

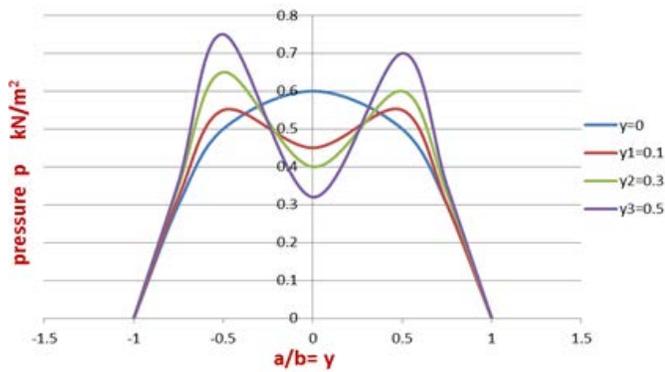


Fig.10 Pressure distribution for different  $a/b=y$

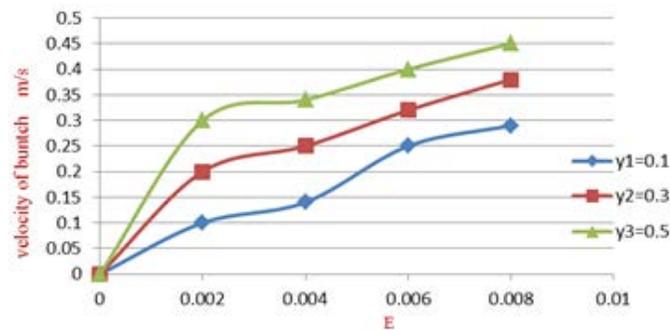


Fig.11 Evaluation of strain rate in the free surfaces comparing with the velocity of the punch

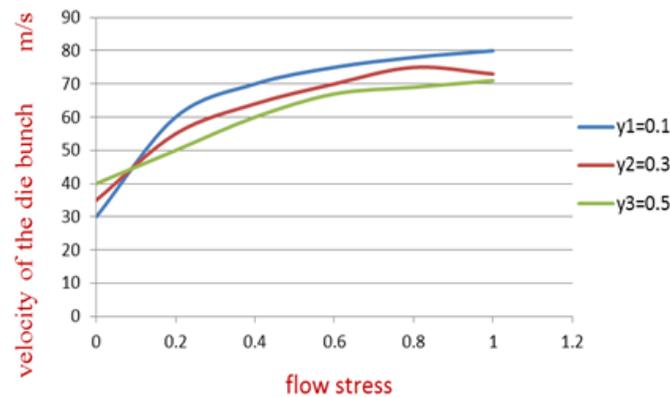


Fig.12 Relationships between the flow stress at 4% strain rate

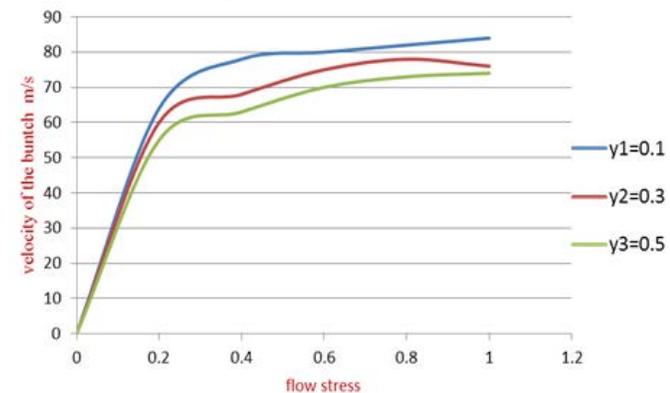


Fig.13 Relationship between the flow stress at 8% strain rate

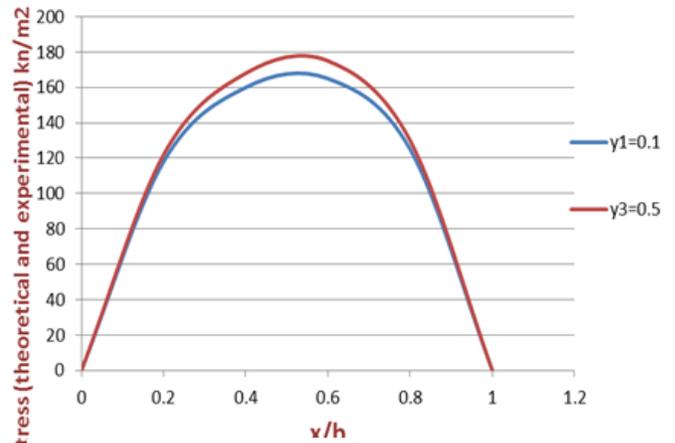


Fig.14(a) Comparison between experimental ( $y_3$ ) and theoretical ( $y_1$ ) distribution of the stresses along the face of the die for  $E=4\%$

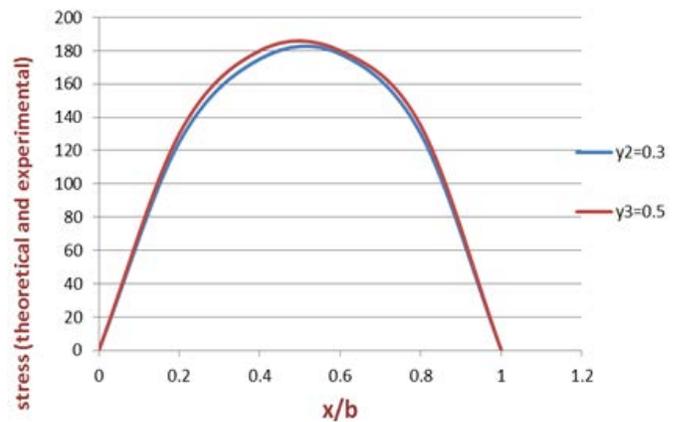


Fig.14(b) Comparison between experimental ( $y_3$ ) and theoretical ( $y_2$ ) distribution of the stresses along the face of the die for  $E=8\%$

## REFERENCES

- [1] Khadem,R and O, Conner J.J, " frictionless Compression of an elastic rectangle between two identical elastic half spaces " ,int.J. Eng. sci., vol 7 page 153-163, 1996.
- [2] M.M. Moshksar and R.Ebrahimi " An analytical approach for backward Extrusion forging of regular hollow components " In. J. mech. sci vol40, No12, pp1247-1263 ,2004.-
- [3] M.M.Moshksar and R.Ebrahimi " Backward Extrusion of axi-symmetric components" processing and fabricasion of adv.Master,sigaphare .pp89-100 nov, 2006.
- [4] M.M.Moshksar, M.H.shariut and M.Bhrami"Upper bound solution for backward –forward extrusion combined Extrusion of the rods and tubes proceeding of IMEC 2004,international mechanical engineering conference p508-520 ,2007.
- [5] Musukelishvili,N.I." Singular Equation (translated by J.R.M.Radok),Noordhoff international publishing, 1980.
- [6] Rao " " principle of metal working "
- [7] S.B.Biner " :A procedure for determination of the strain model techniques " ASME. Trans J.of Evg.indus vol 114,pp90-98 ,2006.
- [8] Sackfield, A and hills, D.A."Sliding contact between dissimilar bodies, J.Tribology,vol 110 ,p 592-598,1998.
- [9] Timoshenko, and Goodier,J.N "Theory of Elasticity" ,2nd edition Mc Graw –Hill ,1951.



**Ban Bakir** (M'1995- SM'2000 F'2006) born in Iraq-Baghdad 1973, B.Sc. Technical university/college of engineering, Mechanical department 1995, M.Sc. University of Baghdad /college of Engineering, Mechanical Department ,Manufacturing processes ,2006 /Iraq. She works as Engineer in laboratories in college of engineering Baghdad university, Lecturer in Baghdad University ,College of Engineering .She has 2 published papers,(Elasto hydrodynamic lubrication analysis of artificial joint with porous material under Quasi static condition) ,( Design and implementation of information system for general diagnostic and remedy),2 published books (statistics) Baghdad Iraq 2008 , (Quality assurance in Higher education) Alkufa /Iraq 2012. Lecturer Ban bakir Alamer, many awards from conference of WIEA held inturke y 2012 ,minister of higher education , Dean of engineering college