# Robust Optimization Approach for Project Selection and Scheduling Problem with Uncertain Activity Duration

Mohammadreza Emami[1], Siamak Haji Yakhchali[2], Ali Namazian [3]

***Abstract***— In this paper, the project selection and scheduling problem is formulated, analyzed, and solved to optimize total expected benefit under uncertain environment. A multi-period project selection and scheduling problem is introduced and modeled by robust optimization approach with uncertain activity duration. Although this integrated problem has become an interesting field of research, the available models suffer from serious shortcomings like uncertainty and this paper proposes a solution to this issue. At first the problem is formulated in certain environment, then by Bertsimas approach uncertainty environment take in to consideration and then the effectiveness of the proposed model is evaluated by the numerical results.

***Keywords***— Project selection, Project scheduling, Robust optimization, Uncertainty

## I. Introduction

PROJECT selection and scheduling decision is one of the main emphases in project management [15]. In real world competitive markets, project selection is one of the most important strategic decisions and is one of the biggest concerns that each enterprise may deal with ([12], [9], and [13]). The economic nature of many projects causes that more benefit can be achieved if a project is completed earlier [4]. If project selection be a smart decision with considering the organization conditions, they cause empowering of organizations in the competitive environment. The project portfolio selection (PPS) problem may consider various objectives but maximum benefit is always considered as a crucial objective.

Many studies have considered this objective function ([4], [12]). Inclusion of project activity scheduling as a sub problem of project portfolio selection helps to improve the overall organization performance even though it increases the complexity of decision making problem. This combined problem is termed as the resource-constrained project portfolio selection and scheduling problem (RCPPSSP). The RCPPSSP can be described as a problem to select an optimal portfolio of projects and schedule them to maximize an organization’s stated objectives.

Early studies used integer linear programming or dynamic techniques to support decision making about project portfolio selection, some studies extended these models and took account of factors that occur in practice, thus they had a more realistic perspective. For example, [5] used a goal programming model for information system project selection, and [11] prepared a multi objective integer optimization model with distributions of costs probability. More recently, new studies were done on project selection and scheduling. [2] introduced a comprehensive model for the portfolio of several objectives, and [15] presented the multiagent evolountiary algorithm for project portfolio selection and scheduling to maximize the NPV. [10] also offered a scenario tree approach for multi-period project selection problems using the real option valuation method. Last but not least, [14] proposed the multi-objective decision analysis for competence oriented project portfolio selection [7]. A review of the project portfolio selection studies revealed that project scheduling was not taken into consideration in a majority of them [6]. Yet, a few studies such [3]. From the perspective of project selection and portfolio scheduling, if we consider only selection aspects and eliminate scheduling, we cannot make efficient use of resources. Therefore, to tackle the problem of simultaneous selection and scheduling, projects need to be prepared with more flexibility with respect to the use of resources and facilities. Recent studies conducted by [15] and [14], therefore, considered new models for project portfolio selection and scheduling. In this study, we have considered selection and scheduling of projects simultaneously in uncertain environment. A summary of the literature review are given in table 1.

In accordance with the above analysis, it can be found that the existing papers rarely consider simultaneously both project portfolio selection and resource constrained project scheduling under uncertain environment. This paper studies the problem under uncertainty .The decision maker aims at selecting a subset of these projects and scheduling their activities. The objective function of the problem is to maximize total benefit. The problem is first mathematically formulated by a mixed integer linear programming model then reformulated with robust optimization approach and

1,Alaodoleh Semnani Institute Of Higher Education, Iran, email:memami84@yahoo.com,09384095478

2, Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran ,email:Yakhchali@ut.ac.ir

**3,** Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran**,** email: a.namazian@ut.ac.ir

then by using numerical example effectiveness of the model is evaluated.

TABLE I
LITERATURE REVIEW OF THE JOINT PROBLEM OF PROJECT SELECTION AND SCHEDULING

| researcher | Year | Solving method | Objective |
|---|---|---|---|
| [13] | 2015 | Ant colony optimization | **NPV-balance resource usage** |
| [15] | 2014 | Multiagent Evolutionary Algorithm | **NPV** |
| [10] | 2014 | Stochastic programming | **NPV** |
| [7] | 2013 | GA-SA-imperialist competitive | **NPV** |
| [8] | 2013 | GA-SA | **NPV** |
| [9] | 2010 | PSO | **NPV-COST-RISK** |
| [4] | 2009 | Imperialist competitive | **NPV** |

## II. PROBLEM DEFINITION AND PROPOSED MATHEMATICAL FORMULATION

The problem of selecting and scheduling projects can be described as follows. There are a set of N projects where each project has its own benefit. To perform each project a set of m activities has to be operated. There are some precedence relationships among the activities of a project. Type of the precedence relationships are assumed to be finish-start with zero time lag, an activity can be started only if all of its predecessors completed and it does not need setup time. This is regardless of the dependencies between the projects. Candidate projects are considered based on their net present values and finally a subset of them are selected. It is assumed that cash inflow is achievable only when a project is finished. A goal is to find a project portfolio such that the net present value of the portfolio is maximized. In the joint problem of project selection and scheduling, a small subset of projects are chosen from the larger set of projects based on the set of desired criteria and the given set of limitations. The first phase of studying such an optimization problem is developing mathematical formulation.

### Mathematical model

In the proposed model, the following indices and parameters are used:

**Parameters:**

N: Total number of available projects

T: Number of time periods (make span)

i: Indices of projects

j; Indices of activities

$d_{ij}$: Duration of the activity *j* from project *i*

$b_{ijt}$: Benefit of the activity *j* from project *i*, in time period *t*

$S_{ij}$: The set of prerequisite activities of the activity *j* from project *i*

**Decision variables**:

$X_{ijt}$: Takes value one if activity *j* from project *i* is finished in time period *t* and otherwise zero.

$Y_i$: Takes value one if project *i* is selected and otherwise zero.

The problem formulation is as follows:

$$max\, E\{\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{t=1}^{T} b_{ijt}X_{ijt}\}$$

St.

$$\sum_{l=1}^{t} X_{ijl} \leq 1 \qquad \forall i,t,\forall j = 1,2,\dots,J_i \tag{1}$$

$$\sum_{l=1}^{T} X_{ijl} = Y_i \quad \forall i,\forall j = 1,2,\dots,J_i \tag{2}$$

$$\sum_{l=1}^{t-1}(t+d_{ij})X_{ijl} \leq \sum_{t=1}^{t-1} tX_{ihl} \quad \forall i = 1,2,\dots,I\,,$$
$$\forall i = 1,2,\dots,I\,, j = 1,2,\dots,J_i - 1\,, ih\epsilon S(ij) \tag{3}$$

$$X_{ijt}\epsilon\{0,1\} \tag{4}$$
$$Y_i\epsilon\{0,1\} \tag{5}$$

The objective function is to maximize the total profit of the selected project portfolio. Constraint (1) ensures that each activity of the projects only at one time can be done. Constraint (2) ensures that all activities of a selected project are completed. It also enforces that every activity in unselected projects is not executed. Constraint (3) describes the precedence relationships among activities which requires an activity to start only after all its predecessors have been completed. Constraints (4) and (5) declare the decision variables.

## III. ROBUST OPTIMIZATION

This robust model in 2004, Developed by Bertsimas and Sim [1], In this model consider the i[th] constraint of the nominal problem or certain model, $a_i'x \leq b_i$. Let $J_i$ be the set of coefficients $a_{ij}$, $j \in j_i$, that are subject to parameter uncertainty i.e., $a_{ij}$, $J_i$ takes values according to a symmetric distribution with mean equal to the nominal value $a_{ij}$ in the interval $[\boldsymbol{a_{ij}} - \boldsymbol{\hat{a}_{ij}},\ \boldsymbol{a_{ij}} + \boldsymbol{\hat{a}_{ij}}]$, For every i, Bertsimas and sim introduced a parameter $\boldsymbol{\Gamma_i}$, not necessarily integer, that takes values in the interval $[\mathbf{0}, |\mathbf{j_i}|]$ , the role of the parameter $\boldsymbol{\Gamma_i}$ is to adjust the robustness of the proposed method against the level of conservatism of the solution. In fact, it is unlikely that all of the $a_{ij}$, $j \in j_i$ will change. Our goal is to be protected against all cases that up to $\lfloor \boldsymbol{\Gamma_i} \rfloor$ of these coefficients are allowed to change, and one coefficient $\boldsymbol{a_{it}}$ changes by $(\boldsymbol{\Gamma_i} - [\boldsymbol{\Gamma_i}])\boldsymbol{a_{it}}$ .In other words, they stipulate that nature will be restricted in its behavior, in that only a subset of the coefficients will change in order to adversely affect the solution. They tried to develop an approach that has the property that if nature behaves like this, then the robust solution will be feasible deterministically, and moreover, even if more than $[\boldsymbol{\Gamma}_i]$ change, then the robust solution will be feasible with very high probability.

Bertsimas and sim consider the following (still nonlinear) formulation:

$$Min\ c'x$$

$$S.t\ \ \textstyle\sum_j a_{ij}x_j + \beta_i(x_j^*, \Gamma_i) \le b_i \quad \forall\, i$$

$$-y_j \le x_j \le y_j \qquad \forall\, j$$

$$l \le x \le u$$

$$x_i \in z \qquad i = 1,2,\dots,k$$

$$\beta_i(x_j^*, \Gamma_i) = \underset{\{S_i\cup\{t_i\}|S_i\subseteq J_0, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}}{Max} \left\{ \sum_{j\in S_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it_i} y_{t_i} \right\}$$

If $\boldsymbol{\Gamma}_i$ is chosen as an integer, the i[th] constraint is protected by $\boldsymbol{\beta}_i(\boldsymbol{x}_j^*, \boldsymbol{\Gamma}_i)$. Note that when $\boldsymbol{\Gamma}_i = \mathbf{0}, \boldsymbol{\beta}_i(\boldsymbol{x}_j^*, \boldsymbol{\Gamma}_i) = \mathbf{0}$ .The constraints are equivalent to that of the nominal problem.by varying $\boldsymbol{\Gamma}_i \in [\mathbf{0}, |\mathbf{j_i}|]$ we have the flexibility of adjusting the robust-ness of the method against the level of conservatism of the solution. In order to reformulate model (4) as a linear optimization model they need the following proposition.

$$\beta_i(x_j^*, \Gamma_i) = \underset{\{S_i\cup\{t_i\}|S_i\subseteq J_0, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}}{Max} \left\{ \sum_{j\in S_i} \hat{a}_{ij}|X_j^*| + (\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it_i}|X_j^*| \right\}$$

Equals the objective function of the following linear optimization problem:

$$\boldsymbol{\beta}_i(\boldsymbol{x}_j^*, \boldsymbol{\Gamma}_i) = \boldsymbol{max}\, \hat{\boldsymbol{a}}_{ij}|\boldsymbol{x}_j|\boldsymbol{z}_{ij}$$

$$\boldsymbol{S.t}\text{:} \sum_j \boldsymbol{z}_{ij} \le \boldsymbol{\Gamma}_i$$

$$\mathbf{0} \le \boldsymbol{z}_{ij} \le \mathbf{1} \quad \forall \boldsymbol{j}$$

Clearly the optimal solution value of the problem (6) consists of $\lfloor \boldsymbol{\Gamma}_i \rfloor$ variables at 1 and one variable at $\boldsymbol{\Gamma}_i - [\boldsymbol{\Gamma}_i]$ this is equivalent to the selection of subset.

Model (4) has an equivalent linear formulation as follows:

$$Min\ c'x$$

$$S.t\text{: } \textstyle\sum_j a_{ij}x_j + \sum_{j\in J_i} p_{ij} + \Gamma_i z_i \le b_i$$

$$z_i + p_{ij} \ge \hat{a}_{ij} y_j \qquad \forall i, j$$

$$p_{ij} \ge 0 \qquad \forall i, j$$

$$z_i \ge 0 \qquad \forall i$$

$$-y_j \le x_j \le y_j \qquad \forall\, j$$

$$l \le x \le u$$

## IV. Robust Optimization Approach In Project Selection And Scheduling Problem

Our uncertain parameter in this research is durations of activities. So based on betsimas and sim model we have:

$$[d_{ij} - \hat{d}_{ij}\xi_{ij}^1, \qquad d_{ij} + \hat{d}_{ij}\xi_{ij}^1]$$

Then we definite $\beta^d(\Gamma^d)$ for our constraint:

$$\sum_{l=1}^{t-1} tX_{ijl} + \sum_{l=1}^{t-1} d_{ij}X_{ijl} - \sum_{t=1}^{t-1} tX_{ihl} + \beta^d(\Gamma^d) \le 0$$

$$\beta^d(\Gamma^d) = \underset{\{s^d\cup\{t^d\}|\ s^d \subseteq t^d, s^d \le \lfloor \Gamma^d \rfloor, t^d \in f^d \setminus s^d\}}{max} \sum_{ij\in s^d} X_{ijl}\, \hat{d}_{ij} + (\Gamma^d - \lfloor \Gamma^d \rfloor)\hat{d}_{tt'} x_{tt'}$$

$\beta^d(\Gamma^d)$ is equal to:

$$\beta^d(X_{ijl}^*, \Gamma^d) = \sum_{ij\in s^d} X_{ijl}\, \hat{d}_{ij}\xi_{ij}^d$$

$$\boldsymbol{S.t}$$

$$\sum_{ij\in f^d} \xi_{ij}^d \le \Gamma^d$$

$$0 \le \xi_{ij}^d \le 1 \qquad \forall ij \in f^d$$

The model is knapsack problem so we have:

$$min \sum_{ij\in f^d} P_{ij}^d + \Gamma^d Q^d$$

$$Q^d + P_{ij}^d \ge \hat{d}_{ij}x_{ijl} \quad \forall ij \in f^d$$

$$Q^d, P_{ij}^d \ge 0 \qquad \forall ij$$

Orally the model of project selection and scheduling problem based on robust optimization approach with uncertain activity duration is:

$$max\ E\{\sum_{i=1}^{I}\sum_{j=1}^{J_i}\sum_{t=1}^{T} P_{ijt}X_{ijt}\}$$

$$\sum_{l=1}^{t} X_{ijl} \le 1 \qquad \forall i, t, \forall j = 1,2,\dots,J_i$$

$$\sum_{l=1}^{T} X_{ijl} = Y_i \quad \forall i, \forall j = 1,2,\dots,J_i$$

$$\sum_{l=1}^{t-1} tX_{ijl} + \sum_{l=1}^{t-1} d_{ij}X_{ijl} - \sum_{t=1}^{t-1} tX_{ihl} + \sum_{ij\in f^d} P_{ij}^d + \Gamma^d Q^d$$

$$\forall i = 1,2,\dots,I\ , j = 1,2,\dots,J_i - 1\ , ih \epsilon S(ij$$

$$Q^d + P_{ij}^d \ge \hat{d}_{ij}X_{ijl} \quad \forall ij \in f^d$$

$$Q^d, P_{ij}^d \ge 0$$

$$X_{ijt} \epsilon \{0,1\}$$

$$Y_i \epsilon \{0,1\}$$

## V. Numerical Examples

For showing the effectiveness of the proposed model, information of a case study based on rafiee's thesis [10], are shown in table 2. There are 20 projects that each project have several activities, benefits and times of the projects are based on time period, each period is 3 months or 90 days. The number of periods is equal to15.

TABLE II
INFORMATION OF PROJECTS

| **Project** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **time** | 4 | 4 | 4 | 3 | 4 | 6 | 2 | 4 | 6 | 4 |
| **benefit** | 8 | 6 | 10 | 3 | 2 | 6 | 8 | 6 | 4 | 8 |
| **activity** | 10 | 5 | 6 | 8 | 3 | 12 | 10 | 5 | 4 | 5 |
| **Project** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| **time** | 4 | 2 | 8 | 4 | 5 | 3 | 4 | 5 | 4 | 4 |
| **benefit** | 6 | 1 | 7 | 9 | 10 | 1 | 5 | 3 | 5 | 6 |
| **activity** | 5 | 6 | 10 | 5 | 6 | 8 | 3 | 12 | 10 | 5 |

## VI. Results

This section gets the results of solving problem in both certain and uncertain models.

To solve the nominal model, we insert all problem data in an Excel Worksheet and link between the Excel Worksheet to export problem data to GAMS. This is implemented on a PC workstation with a 500 MHz CPU and 245 MB memory. The model takes about 2,476 seconds to solve and the optimal objective is equal to 80.

The results of nominal model are shown in table3.

TABLE III
SELECTED PROJECTS IN NOMINAL MODEL

| **Project** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **selection** | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| **Project** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| **selection** | 1 | 1 | 0 | 0 | | 1 | 0 | 0 | 1 | 1 |

In order to analyze the effects of uncertainty, we assume that duration of activities may deviate up to 20% around their nominal values, and we give set some number for $\Gamma^d$ , the robust objective with different values of $\Gamma^d$ is shown in table4.

TABLE IV
ROBUST OBJECTIVES FOR EACH $\Gamma^d$

| $\Gamma^d$ | **Robust objective** | **S** |
|---|---|---|
| **0** | 80 | S1 |
| **0.1** | 79 | S2 |
| **0.2** | 77 | S3 |
| **0.3** | 75 | S4 |
| **0.4** | 75 | S5 |
| **0.5** | 73 | S6 |
| **0.6** | 72 | S7 |
| **0.7** | 68 | S8 |
| **0.8** | 66 | S9 |
| **0.9** | 64 | S10 |
| **1** | 62 | S11 |

Also the results of the selected projects are shown in table5.

TABLE V
SELECTED PROJECTS FOR EACH $\Gamma^d$

| **p** | **Selected projects in each robust objective** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 | S11 |
| **1** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **2** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **3** | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **6** | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| **7** | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| **8** | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| **9** | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **10** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| **11** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **12** | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| **13** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **14** | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **15** | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| **16** | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| **17** | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **18** | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| **19** | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **20** | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |

It is intuitive that the robust objective is always a decreasing function of $\Gamma^d$. Also start times of each project that selected in each robust objective are shown in table 5, as mentioned earlier, times are based on time period.

TABLE IV
START TIMES OF SELECTED PROJECTS

| **p** | **Start times of selected projects in each robust objective** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\Gamma^d$ | | | | | | | | | | |
| | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 | S10 | S11 |
| **1** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| **2** | 5 | 6 | 6 | 6 | 5 | 6 | 6 | 6 | 6 | 8 | 6 |
| **3** | 9 | 9 | - | 8 | 8 | 9 | 8 | 8 | 10 | 9 | 8 |
| **4** | - | - | - | - | - | - | - | - | - | - | - |
| **5** | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 |
| **6** | 1 | 2 | 1 | 2 | 2 | - | - | - | 1 | 1 | 2 |
| **7** | 7 | 8 | 7 | 7 | - | - | - | - | 7 | 9 | 8 |
| **8** | 10 | 10 | - | 10 | 9 | 9 | | - | - | - | - |
| **9** | - | - | 1 | - | - | - | - | - | - | - | - |
| **10** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| **11** | 6 | 6 | 6 | 5 | 6 | 5 | 6 | 6 | 5 | 5 | 4 |
| **12** | 1 | 1 | 1 | 1 | - | 1 | - | 2 | 1 | - | 1 |
| **13** | - | - | - | - | - | - | - | - | - | - | - |
| **14** | - | - | 4 | - | - | - | - | - | - | - | - |
| **15** | 7 | 6 | 10 | - | 10 | 8 | | - | - | - | - |
| **16** | 5 | - | - | 5 | 5 | - | - | 6 | 5 | 5 | 4 |
| **17** | - | - | 1 | - | - | - | - | - | - | - | - |
| **18** | - | - | 5 | - | - | 5 | 4 | - | - | - | - |
| **19** | 10 | 11 | - | 11 | 10 | 10 | 11 | 12 | 10 | 10 | 10 |
| **20** | 1 | 1 | 1 | 1 | 1 | - | - | 2 | - | - | - |

## VII. CONCLUSION

This paper studied the problem of project selection and scheduling under uncertain environment. Each project requires a set of activities. There is a given time limitation and the decision maker should select and schedule a subset of available projects that maximize the total profit under uncertain environment. We first formulate the problem by integer linear programming model. Then by Robust optimization approach based on Bertsimas and sim techniques, formulate the model under uncertainty. The results showed that the proposed model is effective for organizations in the case of decision about project selection problem based on project scheduling in uncertain environment.

## REFERENCES

[1] Dimitris Bertsimas., Melvyn Sim. "The price of Robustness." *Operations research*, 2004.
[2] F. Carazo, *et al.*, "Solving a comprehensive model for multiobjective project portfolio selection," *Computers & operations research,* vol. 37, pp. 630-639, 2010.
[3] Ghasemzadeh, F.,Archer, N., Iyogun., "A zero-one model for project portfolio selection and scheduling." Journal of the Operational Research Society,Vol50,Number7, 1999, pp.745-755.
[4] J. Chen and R. G. Askin, "Project selection, scheduling and resource allocation with time dependent returns," *European Journal of Operational Research*, vol. 193, no. 1, pp. 23–34, 2009.
[5] M. A. Badri, *et al.*, "A comprehensive 0–1 goal programming model for project selection," *International Journal of Project Management,* vol. 19, pp. 243-252, 2001.
[6] M. A. Coffin and B. W. Taylor III, "Multiple criteria R&D project selection and scheduling using fuzzy logic," *Computers and Operations Research*, vol. 23, no. 3, pp. 207–220, 1996.
[7] Naderi, Bahman., "The Project Portfolio Selection and Scheduling Problem: Mathematical Model and Algorithms." Journal of Optimization in Industrial Engineering,Vol13 , 2013, pp.65-72.
[8] Nikkhahnasab, Mostafa., Najafi,Amir Abbas., "Project Portfolio Selection with the Maximization of Net Present Value." Journal of Optimization in Industrial Engineering,Vol17, 2013, pp.85-92.
[9] Rabbani, M., Aramoon Bajestani,M., Baharian Khoshkhou,G., "A multi-objective particle swarm optimization for project selection problem." Expert Systems with Applications, 2010,pp.315–321.
[10] Rafiee, Majid., Kianfar,Farhad., Farhadkhani,Mehdi., "A multistage stochastic programming approach in project selection and scheduling." The International Journal of Advanced Manufacturing technology, 2014, pp.2125-2137.
[11] S. A. Gabriel, *et al.*, "A multiobjective optimization model for project selection with probabilistic considerations," *Socio-Economic Planning Sciences*, vol. 40, pp. 297-313, 2006.
[12] S. Ghorbani and M. Rabbani, "A new multi-objective algorithm for a project selection problem," *Advances in Engineering Software*, vol. 40, pp. 9-14, 2009.
[13] Tofighian, Ali Asghar., Bahman,Naderi., "Modeling and solving project selection and scheduling problem." Computers & Industrial Engineering, 14 January 2015.
[14] W. J. Gutjahr, S. Katzensteiner, P. Reiter, C. Stummer, and M. Denk, "Competence-driven project portfolio selection, scheduling and staff assignment," *Central European Journal of Operations Research*, vol. 16, no. 3, pp. 281–306, 2008.
[15] Y. Shou and Wenwen Xiang, Ying Li, andWeijian Yao, "A multi agent Evoluntiary algorithm for the resource Constrained project portfolio selection and scheduling problem", *Mathematical Problems in Engineering*, 2014.
[16] Xiaoxia Huang, Tianyi Zhao. "Project selection and scheduling with uncertain net income and investment cost." *Applied Mathematics and Computation 247*, 2014: pp.61-71.