The Multi-Item Capacitated Lot-Sizing Problem With Safety Stocks In Closed-Loop Supply Chain

Esmaeil Mehdizadeh, and Amir Fatehi Kivi

***Abstract*—** This paper proposes a new mixed integer programming model for multi-item capacitated lot-sizing problem with setup times, safety stock and demand shortages in closed-loop supply chain the returned products from customers can either be disposed or be remanufactured to be sold as new ones again. The problem is NP-hard and to solve it, a simulated annulling approach is used. To verify and validate the efficiency of the SA algorithm, the results are compared with those of the Lingo 8 software. Results suggest that the SA algorithm have good ability of solving the problem, especially in the case of large and medium-sized problems for which Lingo 8 cannot produce solutions.

***Keywords*—** Closed-loop supply chain, Lot-sizing, Safety stocks, Simulated annealing.

I. INTRODUCTION

THE production planning problems encountered in real-life situations are generally intractable due to a number of practical constraints. The decision maker has to find a good feasible solution in a reasonable execution time rather than an optimal one. Their main idea was to find relationships between the performance of the heuristic and the computational burden involved in finding the solution. Chen and Thizy proved that the multi-item capacitated lot-sizing problem with setup times is strongly NP-hard. There are many references dealing with the capacitated lot-sizing problem and to explain why one of the most popular among exact and approximate solution methods used lagrangian relaxation of the capacity constraint and compare this approach with every alternate relaxation of the classical formulation of the problem[1]. Absi and Kedad-Sidhoum addressed a multi-item capacitated lot-sizing problem with setup times, safety stock and demand shortages. They proposed a Lagrangian relaxation of the resource capacity constraints and developed a dynamic programming algorithm to solve the problem [2]. Süral et al. considered a lot-sizing problem with setup times where the objective is to minimize the total inventory carrying cost only. They proposed some efficient Lagrangian relaxation based heuristics for a lot-sizing problem [3]. Chu et al. addressed a real-life production planning problem arising in a manufacturer of luxury goods. This problem can be modeled as a single-item dynamic lot-sizing model with backlogging, outsourcing and inventory capacity. They showed that this problem can be solved in $O(T^4 \log T)$ time where T is the number of periods in the planning horizon [4]. Golany et al. studied a production planning problem with remanufacturing. They proved the problem is NP-complete and obtain an $O(T^3)$ algorithm for solve the problem [5]. Li et al. analyzed a version of the capacitated dynamic lot-sizing problem with substitutions and return products. They first applied a genetic algorithm to determine all periods requiring setups for batch manufacturing and batch remanufacturing, and then developed a dynamic programming approach to provide the optimal solution to determine how many new products are manufactured or return products are remanufactured in each of these periods [6]. Pan et al. addressed the capacitated dynamic lot- sizing problem arising in closed-loop supply chain where returned products are collected from customers. They assumed that the capacities of production, disposal and remanufacturing are limited, and backlogging is not allowed. Moreover, they proposed a pseudo-polynomial algorithm for solving the problem with both capacitated disposal and remanufacturing [7]. Zhang et al. investigated the capacitated lot-sizing problem in closed-loop supply chain considering setup costs, product returns, and remanufacturing. They formulated the problem as a mixed integer program and propose a Lagrangian relaxation-based solution approach [8]. Tang provides a brief presentation of simulated annealing techniques and their application in lot-sizing problems [9].

The main contribution of this paper is twofold. First, we develop the multi-item capacitated lot-sizing problem with demand shortage, safety stock, deficit costs, capacity stock and several manners in closed-loop supply chain where returned products are collected from customers. Then we design simulated annealing (SA) algorithm to solve the problem.

II. MATHEMATICAL FORMULATION

In this section, we present an MIP formulation of the problem. In order to close the gap between the conditions of

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the problem and the real world conditions in this research, the multi-item lot-size problem has been studied with considerations of production line equilibrium limitation and capacity limitation. Not only has there been a consideration of different production manners for products, but also the model has been designed in the conditions of having safety stock and shortage being allowed. Also the factory is responsible for processing used products returned from customers. Two options are available for these returned products: remanufacturing and disposal. Remanufactured products can be sold as new ones with the same quality commitment [7].The main goal is to present a mathematical model to optimize production, inventory, outsourcing, shortage, remanufactured and disposal quantities as well as determine the best production manner.

A. Assumptions

Before the formulation is considered, the following assumptions are made on the problem:

I. The demand is considered deterministic.
II. The amount of the returned products is regarded deterministic over the planning horizon.
III. Shortage is backlogged.
IV. Shortage and inventory costs must be taken into consideration at the end.
V. Raw material resource with given capacities are considered.
VI. The quantity of inventory and shortage at the beginning of the planning horizon is zero.
VII. The quantity of inventory and shortage at the end of the planning horizon is zero.

B. Parameters

T: Number of periods, indexed from 1 to T, involved in the planning horizon
N: Number of products, i = 1, …, N
J: Number of production manner, j = 1,…, J
d_{it}: The demand for product i in the period t
L_{it}: The quantity of the safety stock of product i in the period t
r_{it}: The selling price per unit of product i in the period t
C_{ijt}: The production cost of each unit of product i in the period t through the manner j
A_{ijt}: The setup cost of the production of product i in the period t through the manner j
h_{it}^{+} : The unit holding cost of product i in the period t
h_{it}^{-} : Unitary safety stock deficit cost of product i in period t
∂_{it} : Unitary shortage cost of product i in period t
B_{kt}: The capacity of the K source at hand in the period t
α_{ik}: The quantity of the K source used by each unit of the product i
f_{ijk}: The quantity of wasted K source for product i produced through the manner j
γ_{it} : Unit out-sourcing cost of each unit of product i in the period t
M_i : A large number
τ_{ik} : The K source consumption for repair of item i
v_i : Space needs for per unit of product i
φ_t : The total available space in period t
F_{it}: The cost of disposing returned products for each unit of product i in period t
g_{it}: The cost of remanufacturing returned products for each unit of product i in period t
θ_{it} :The unit holding cost of product i of returned products in period t
C_{it}^{d} :The maximum number of returned products of product i that could be disposed in period t
C_{it}^{r} :The maximum number of returned products of product i that could be remanufactured in period t
R_{it}: the number of returned products of product i in period t

Decision Variables

X_{ijt}: Production quantity for product i in the period t through the manner j
X_{it}^{f} :The number of returned products of product i that remanufactured in period t
X_{it}^{s} :The number of returned products of product i that disposed in period t
y_{ijt}: Binary variable; 1 if the product i is produced in the period t through the manner j, otherwise $y_{ijt}=0$
U_{it}: Out-sourcing level of product i in the period t
I_{it}^{r} :The number of returned products of product i held that in inventory at the end of period t
I_{it}^{-} : The quantity of shortage of product i in the period t
S_{it}^{+} : The quantity of overstock deficit of product i in the period t
S_{it}^{-} : The quantity of safety stock deficit of product i in the period t

The objective function (1) shows difference between selling price with the total cost. Constraints (2) are the inventory flow conservation equations through the planning horizon. Constraints (3) and (4) define respectively, the demand shortage and the safety stock deficit for item *i* at end period is zero. Constraints (5) are the inventory flow conservation equations for returned products. Constraints (6) are the capacity constraints; the overall consumption must remain lower than or equal to the available capacity.

$$MaxZ=\sum_{t=1}^{T}\left(\sum_{i=1}^{N}\left(r_{it}(\sum_{j=1}^{J}X_{ijt}+X_{it}^{s}+U_{it})-\sum_{j=1}^{J}(C_{ijt}X_{ijt}+A_{ijt}y_{ijt})-\partial_{it}I_{it}^{-}-h_{it}^{+}S_{it}^{+}-h_{it}^{-}S_{it}^{-}-\gamma_{it}U_{it}-F_{it}X_{it}^{f}-g_{it}X_{it}^{s}-\theta_{it}I_{it}^{r}\right)\right) \quad (1)$$

s.t: (2)

$$S_{i,t-1}^{+}-S_{i,t-1}^{-}-I_{i,t-1}^{-}+I_{i,t}^{-}+\sum_{j=1}^{J}X_{ijt}+X_{it}^{s}+U_{it}=S_{it}^{+}-S_{it}^{-}+d_{it}+L_{it}-L_{i,t-1}\ \forall\ i=1,2,....,N\ ,\ t=1,2,....,T$$

$$S_{iT}^{+}=0 \quad (3)$$

$$I_{i,T}^{-}=0 \quad (4)$$

$$I_{it}^{r}=I_{i,t-1}^{r}-X_{it}^{f}-X_{it}^{s}+R_{it}\ \forall\ t=1,2,....,T\ ,\quad i=1,2,....,N \quad (5)$$

$$\sum_{i=1}^{N}\left(\sum_{j=1}^{J}(\alpha_{ik}X_{ijt}+f_{ijk}y_{ijt})+\tau_{ik}X_{it}^{s}\right)\le B_{kt}\ \forall\ k=1,2,....,K\ ,\ t=1,2,....,T \quad (6)$$

$$X_{ijt}\le My_{ijt}\ \forall\ j=1,2,....,J\ ,\ i=1,2,....,N\ ,\ t=1,2,....,T \quad (7)$$

$$I_{it}^{-}\le d_{it}\ \forall\ i=1,2,....,N\quad ,\ t=1,2,....,T-1 \quad (8)$$

$$S_{it}^{-}\le L_{it}\ \forall\ i=1,2,....,N\quad ,\ t=1,2,....,T \quad (9)$$

$$0\le U_{it}\le I_{i,t-1}^{-}+S_{i,t-1}^{-}+d_{it}+L_{it}\ \forall\ i=1,2,....,N\quad ,\ t=1,2,....,T \quad (10)$$

$$X_{it}^{f}\le C_{it}^{d}\ \forall\ i=1,2,....,N\quad ,\ t=1,2,....,T \quad (11)$$

$$X_{it}^{s}\le C_{it}^{r}\ \forall\ i=1,2,....,N\quad ,\ t=1,2,....,T \quad (12)$$

$$\sum_{i=1}^{N}v_{i}\left(\sum_{j=1}^{J}X_{ijt}+X_{it}^{S}+U_{it}\right)\le\varphi_{t}\ \forall\ t=1,2,....,T \quad (13)$$

$$y_{ijt}\in\{0,1\}\ \forall\ i=1,2,....,N\ ,\ j=1,2,....,J\ ,\quad t=1,2,....,T \quad (14)$$

$$X_{ijt},X_{it}^{f},X_{it}^{s},I_{it}^{r},I_{it}^{-},S_{it}^{-},S_{it}^{+}\ge 0\ \forall\ i=1,2,....,N\quad ,j=1,2,....,J\ ,t=1,2,....,T \quad (15)$$

If we produce an item *i* at period *t*, then constraints (7) impose that the quantity produced must not exceed a maximum production level M_{it}. M_{it} could beset to the minimum between the total demand requirements for item *i* on section [*t*, *T*] of the horizon and the highest quantity of item *i* that could be produced regarding the capacity constraints, M_{it} is then equal Eq. (16) to:

$$M_{it}=Min\left(\frac{B_{kt}-f_{ijk}}{\alpha_{ijk}},\sum_{t=1}^{T}d_{it}\right) \quad (16)$$

Constraints (8) and (9) define upper bounds on, respectively, the demand shortage and the safety stock deficit for item *i* in period *t*. Constraints (10) ensure that outsourcing level U_{it} at period t is nonnegative and cannot exceed the sum of the demand, safety stock of period t and the quantity backlogged, safety stock deficit from previous periods. Constraints (11) and (12) are the capacity constraints of disposal, remanufacturing. Constraints (13) are the Maximum space available for storage of items in excess. Constraints (14) and (15) characterize y_{ijt} is a binary variable and the variable's domains: $X_{ijt},X_{it}^{f},X_{it}^{s},I_{it}^{r},I_{it}^{-},S_{it}^{-},S_{it}^{+}$ are non-negative for $i \in$ N, $j \in$ J and $t \in$ T.

III. Simulated Annealing Algorithm

Simulated annealing (SA) is one of the meta-heuristic algorithms which initially presented by Kirkpatrick et al. [10]. The SA methodology draws its analogy from the annealing process of solids. The steps of implementing of SA algorithm to solve the problem are presented in below:

Step 1- Representation schema: In this paper, the general structure of the solution representation performed for running the simulated annealing for four periods of two products with one production manner is shown in Fig. 1.

Chromosome1= Product number × Number of periods × Number of production manner

Chromosome2= Product number × Number of periods

Chromosome 1	Y_{111}	Y_{112}	Y_{113}	Y_{114}	Y_{211}	Y_{212}	Y_{213}	Y_{214}
	0	1	1	0	0	1	1	0

Chromosome 2	X^S_{11}	X^S_{12}	X^S_{13}	X^S_{14}	X^S_{21}	X^S_{22}	X^S_{23}	X^S_{24}
	2	2	0	1	3	0	1	2

Fig. 1 Solution representation

Step 2- Neighborhood scheme: At each temperature level a search process is applied to explore the neighborhoods of the current solution. In this paper we use mutation scheme, Fig. 2 illustrates this operation on the each of four periods of two products with one production manner.

Chromosome 1	Y_{111}	Y_{112}	Y_{113}	Y_{114}	Y_{211}	Y_{212}	Y_{213}	Y_{214}
	0	1	1	0	0	1	1	0

Chromosome 1	Y_{111}	Y_{112}	Y_{113}	Y_{114}	Y_{211}	Y_{212}	Y_{213}	Y_{214}
	0	1	0	0	0	1	1	0

Chromosome 2	X^S_{11}	X^S_{12}	X^S_{13}	X^S_{14}	X^S_{21}	X^S_{22}	X^S_{23}	X^S_{24}
	2	2	0	1	3	0	1	2

Chromosome 2	X^S_{11}	X^S_{12}	X^S_{13}	X^S_{14}	X^S_{21}	X^S_{22}	X^S_{23}	X^S_{24}
	1	0	3	2	0	3	2	1

Fig. 2 An example of the neighborhood structure

Step 3- Cooling schedule scheme: Initially, T is set to a high value, T_i, and it can be reduced with some patterns at each step of algorithm. The cooling schedule with $T_i = \alpha \times T_{i-1}$ (where α is the cooling factor constant $\alpha \in (0, 1)$ is considered as cooling pattern for this research.

Step 4- Termination condition: The SA continues the above procedure until the termination condition is satisfied ($T < T_F$). Initial and final temperatures have pre-determined constant set.

Remark: because in this model objective function is type of maximum, we have to multiply our objective function in the negative.

IV. Experimental Results

We try to test the performance of the SA in finding good quality solutions in reasonable time for the problem. For this purpose, 15 problems with different sizes are generated. These test problems are classified into three classes: small size, medium size and large size multi-item capacitated lot-sizing problem with safety stocks in closed-loop supply chain problems. The number of products, production manners and periods has the most impact on problem hardness. The proposed model coded with Lingo (ver.8) software using for solving the instances and the SA is implemented to solve each instance in five times to obtain more reliable data. For implementation SA algorithm, the parameters set as: T_0 = 1000, α=0.99, L=50 and T_f =0. The best results are recorded as a measure for the related problem. Table 1 shows details of computational Selection: Highlight all author and affiliation lines.

TABLE I.
Details Of Computational Results For All Test Problems

					Objective Function Value (OFV)			
No	Class	Product	manner	Period	Lingo		SA	
					OFV	Time	OFV	Time
1	Small	2	2	3	1366842	0	1366842	6
2		2	2	5	2170470	0	2165482	26
3		2	3	5	2180288	1	2165482	31
4		5	3	5	4692306	100	4286763	62
5		5	2	6	5497372	300	5037989	75
6	Medium	5	3	6	5621934	750	4997012	341
7		5	4	7	6832705	1011	5950741	369
8		5	3	8	-------	-------	6754581	388
9		5	4	8	-------	-------	10134054	400
10		6	3	9	-------	-------	8551200	680
11	Large	5	3	10	-------	-------	8624198	700
12		6	3	11	-------	-------	9571335	735
13		6	4	12	-------	-------	10601543	760
14		6	3	12	-------	-------	19109673	850
15		6	4	12	-------	-------	11650807	900

— Means that a feasible solution has not been found after 3600 s of computing time.

$$r_{it} \in [50000, 70000], C_{ijt} \in [60, 95], A_{ijt} \in [16000, 30000], d_{it} \in [1,8], \partial_{it} \in [12,18]$$
$$h_{it}^{+} \in [7,11], h_{it}^{-} \in [12,17], B_{kt} = 12, \varphi_t = 20, V_i = 1, \alpha_{ijk} = 1, \mathrm{f}_{ijk} = 1, L_{it} \in [1,4]$$
$$F_{it} \in [65,85], F_{it} \in [65,85], g_{it} \in [15,20], \theta_{it} \in [7,9], C_{it}^{d} \in [1,2], C_{it}^{r} \in [1,3], R_{it} \in [1,3]$$

The results of running SA is compared with the optimal solution of the instances, obtained from Lingo software, in row 1 to 7 of Table 1. Comparing the CPU times of exact solution in the sixth column of Table 1 confirms that computation time grows exponentially by increasing the dimension of the problem Also, after the seventh row the Lingo software has not reached the optimal solution after 3600s of computation time. A general review of the results shows that:

- The SA has the ability to obtain solution for all test problems.
- The SA can find good quality solutions for medium and large size problems.
- The objective values obtained by SA are closes to Lingo results

V. Conclusion

In this paper, we propose a mathematical formulation for a new multi-item capacitated lot-sizing problem with setup times in closed-loop supply chain. This formulation takes into account several industrial constraints such as shortage costs, safety stock deficit costs, limited outsourcing and return products. Due to the complexity of the problem, SA algorithm is used to solve problem instances. Several problems with different sizes generated and solved by SA and Lingo software. The results show that the SA algorithm able to find good quality solutions in reasonable time. One straightforward opportunity for future research is extending the assumption of the proposed model for including real conditions of production systems such as limited inventory, fuzzy demands and etc. Also, developing a new heuristic or metaheuristic to construct better feasible solutions.

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